Chapter 10 Transmission of Digital Information via Carrier Modulation (I)

Quadrature Amplitude-Modulated Digital Signals (1/8)

- The transmitted signal waveforms of quadrature amplitude modulation (QAM) signals have the form

$$u(t) = A_{mc} g_T(t)\cos(2\pi f_c t) + A_{ms} g_T(t)\sin(2\pi f_c t), \quad m = 1,2,\ldots, M,$$

where $\{A_{mc}\}$ and $\{A_{ms}\}$ are the sets of amplitude levels that are obtained by mapping k-bit sequences into signal amplitudes. (A_{mc}, A_{ms}) is the coordinate of a constellation point

- Note the in-phase component $A_{mc} g_T(t)\cos(2\pi f_c t)$ and the quadrature component $A_{ms} g_T(t)\sin(2\pi f_c t)$ of $u(t)$ are mutually orthogonal in the inner-product space defined by

$$\langle f(t), h(t) \rangle \stackrel{\Delta}{=} \int_0^T f(t)h(t)dt$$

Quadrature Amplitude-Modulated Digital Signals (2/8)

- In general, rectangular signal constellations result when two quadrature carriers are each modulated by PAM
- Fig. 10.18 illustrate a 16-QAM signal constellation that is obtained by amplitude modulating each quadrature carrier by M=4 PAM

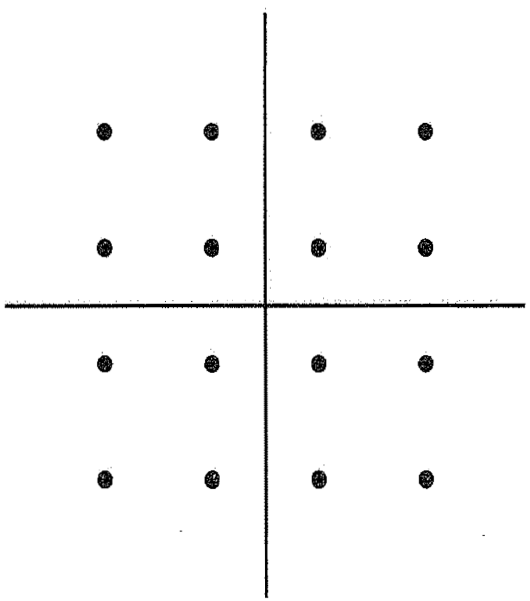

Figure 10.18 $M = 16$-QAM signal constellation.

Quadrature Amplitude-Modulated Digital Signals (3/8)

- Fig. 10.19 illustrates the modulator for QAM implementation

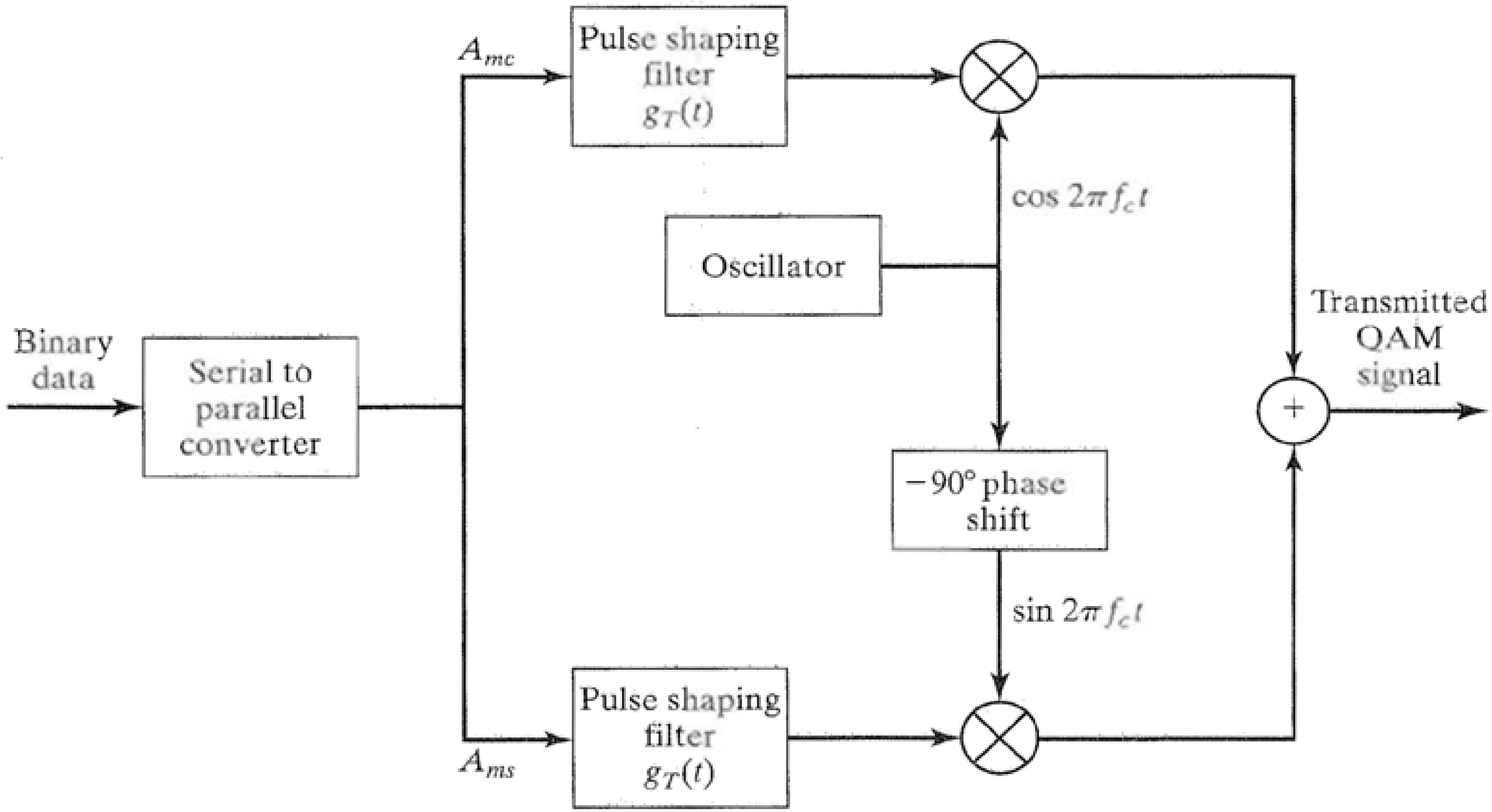

Figure 10.19 Functional block diagram of a modulator for QAM.

Quadrature Amplitude-Modulated Digital Signals (4/8)

- QAM may be viewed as a form of combined digital-amplitude and digital-phase modulation. Thus, the transmitted QAM signal waveforms may be expressed as

$$u(t) = A_m g_T(t)\cos(2\pi f_c t + \theta_n), \quad m = 1,2,\ldots, M_1, n = 1,2,\ldots, M_2$$

- Let R_b denote the transmission bit rate. If $M_1 = 2^{k_1}$ and $M_2 = 2^{k_2}$, the combined amplitude- and phase-modulation method results in the simultaneous transmission of $k_1 + k_2 = \log_2 M_1 M_2$ bits occurring at a symbol rate $R_b / (k_1 + k_2)$

Quadrature Amplitude-Modulated Digital Signals (5/8)

- It is clear that the geometric signal representation of $u(t)$ is in the form of two-dimensional signal vectors

$$\mathbf{s}_m = (\sqrt{\mathcal{E}_s} A_{mc}, \sqrt{\mathcal{E}_s} A_{ms}),\ m=1,2,\ldots,M$$

and the orthogonal basis functions as

$$\psi_1(t) = \sqrt{\frac{1}{\mathcal{E}_s}} g_T(t) \cos(2\pi f_c t)$$

$$\psi_2(t) = \sqrt{\frac{1}{\mathcal{E}_s}} g_T(t) \sin(2\pi f_c t)$$

- $\mathcal{E}_s$ is not necessarily equal to the average symbol energy $\mathcal{E}_{av}$; however, it does when the average length of signal vectors (A_{mc}, A_{ms}), $m=1,2,\ldots,M$, is normalized

Quadrature Amplitude-Modulated Digital Signals (6/8)

- It should be noted that M=4 rectangular QAM and M=4 PSK have identical signal constellations. Examples of signal-space constellations for QAM are shown in Fig. 10.20(a)

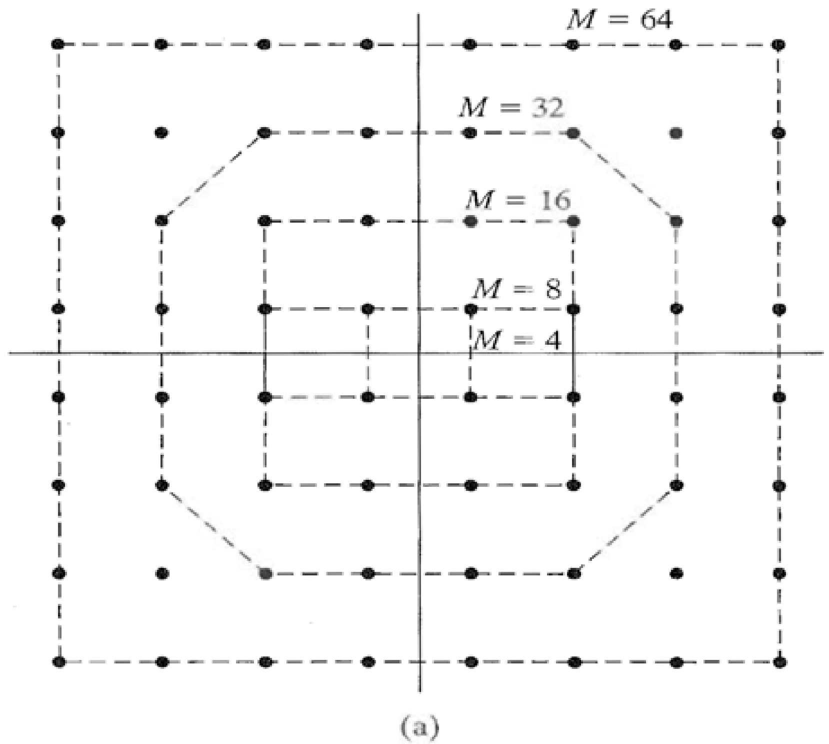

Figure 10.20 (a) Rectangular signal-space constellations for QAM. (b, c) Examples of combined PAM–PSK signal-space constellations.

Quadrature Amplitude-Modulated Digital Signals (7/8)

- Examples of combined PAM-PSK signal-space constellations are shown in Fig. 10.20(b) and Fig. 10.20(c)

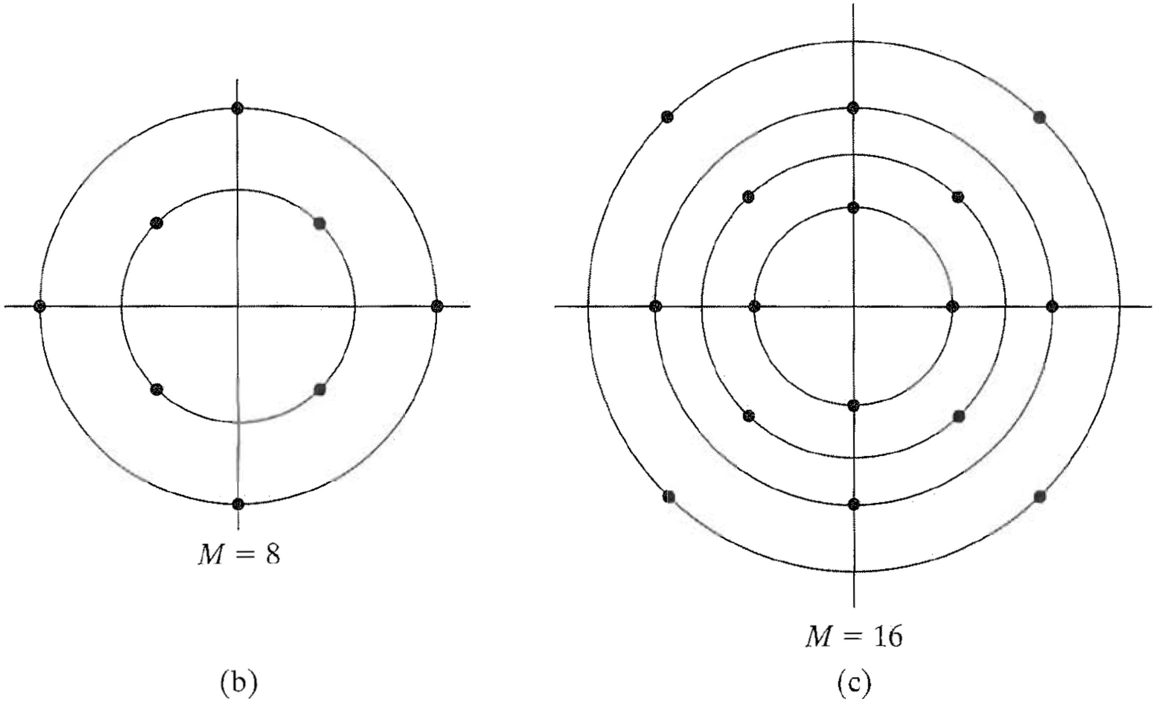

Figure 10.20 (a) Rectangular signal-space constellations for QAM. (b, c) Examples of combined PAM–PSK signal-space constellations.

Quadrature Amplitude-Modulated Digital Signals (8/8)

- The average symbol energy for those signal constellations is simply the sum of the average energies in the quadrature carriers
- The average energy per symbol is given as

$$\mathcal{E}_{av} = \frac{1}{M}\sum_{m=1}^{M}\left\|\mathbf{s}_m\right\|^2.$$

The notation $\left\| \cdot \right\|^2$ denotes squared Eulidean distance

- The distance between any pair of signal points is

$$d_{mn} = \left\|\mathbf{s}_m - \mathbf{s}_n\right\|$$

Demodulation and Detection of Quadrature Amplitude-Modulated Signals (1/3)

- The received signal $r(t)$ corrupted by additive Gaussian noise can be expressed as

$$r(t) = A_{mc} g_T(t)\cos(2\pi f_c t + \phi) + A_{ms} g_T(t)\sin(2\pi f_c t + \phi) + n(t)$$

- Suppose that an estimate $\hat{\phi}$ of the carrier phase is available at the demodulator. Then, the received signal may be correlated with the two basis functions

$$\psi_1(t) = \sqrt{\frac{1}{\mathcal{E}_s}} g_T(t)\cos(2\pi f_c t + \hat{\phi})$$

and

$$\psi_2(t) = \sqrt{\frac{1}{\mathcal{E}_s}} g_T(t)\sin(2\pi f_c t + \hat{\phi})$$

Demodulation and Detection of Quadrature Amplitude-Modulated Signals (2/3)

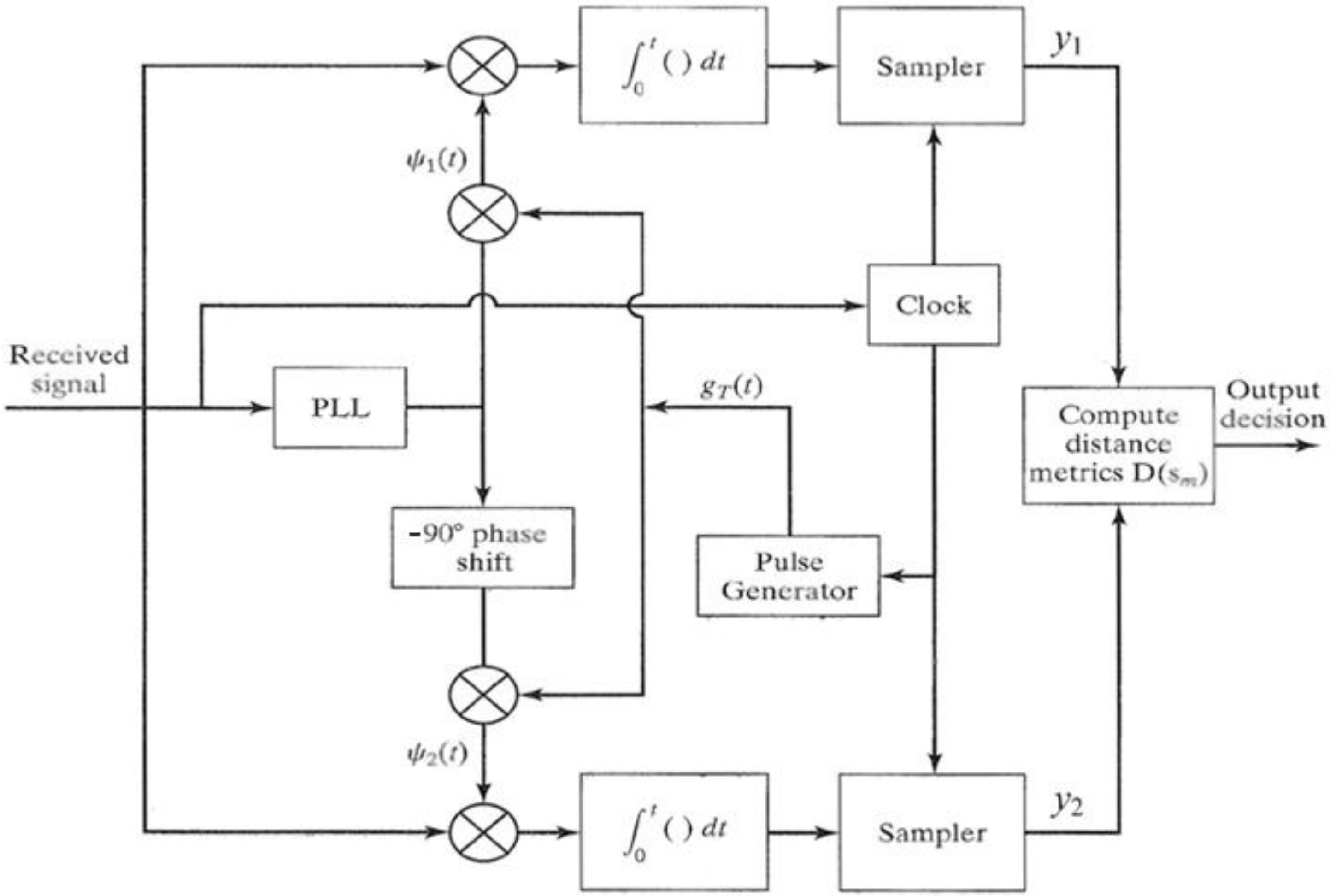

Figure 10.22 Demodulation and detection of QAM signals.

Demodulation and Detection of Quadrature Amplitude-Modulated Signals (3/3)

- Define $y=(y_1,y_2)$. The optimum detector computes the distance metrics

$$D(y,\mathbf{s}_m)=\left\| y\text{-}\mathbf{s}_m \right\|^2, \quad m=1,2,\ldots,M,$$

and selects the signal corresponding to the smallest value of $D(y,\mathbf{s}_m)$. If a correlation metric is used in place of a distance metric, it is important to recognize that correlation metrics must employ bias correction ($i.e., \left\| \mathbf{s}_m \right\|^2$) because the QAM signals are not equal energy

Probability of Error for QAM (1/8)

- We consider two QAM signal sets. The first is a four-phase modulated signal and the second is a QAM signal with two amplitude levels. For the four-phase signal with $(A_{mc},A_{ms})=(\pm 1,\pm 1)$, we have

$$\mathcal{E}_{av}=\tfrac{1}{4}\cdot 4\cdot 2\mathcal{E}_s=2\mathcal{E}_s$$

- We impose the condition that $d_{min}=2\sqrt{\mathcal{E}_s}$

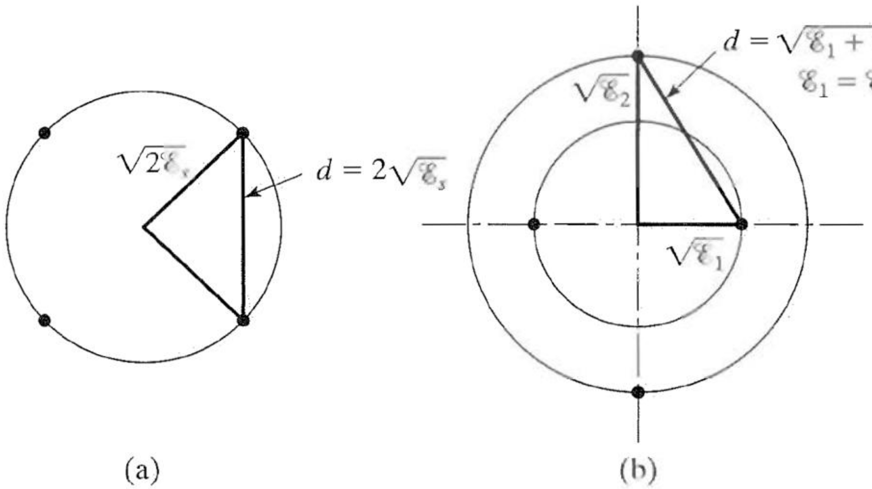

Figure 10.23 Two four-point signal constellations.

Probability of Error for QAM (2/8)

- For the two-amplitude, four-phase QAM, the average energy is

$$\mathcal{E}_{av} = \tfrac{1}{4}[2 \cdot 3\,\mathcal{E}_s + 2 \cdot \mathcal{E}_s] = 2\mathcal{E}_s,$$

which is the same average energy as the M=4-phase signal constellation. Here $\sqrt{\mathcal{E}_s}$ acts as a scaling factor for transforming constellation coordinate into average symbol energy

- Next, consider M=8 QAM. In this case, there are many possible signal constellations. The four signal constellations shown in Fig. 10.24 consist of two amplitudes and have a minimum distance between signal points of $2\sqrt{\mathcal{E}_s}$

Probability of Error for QAM (3/8)

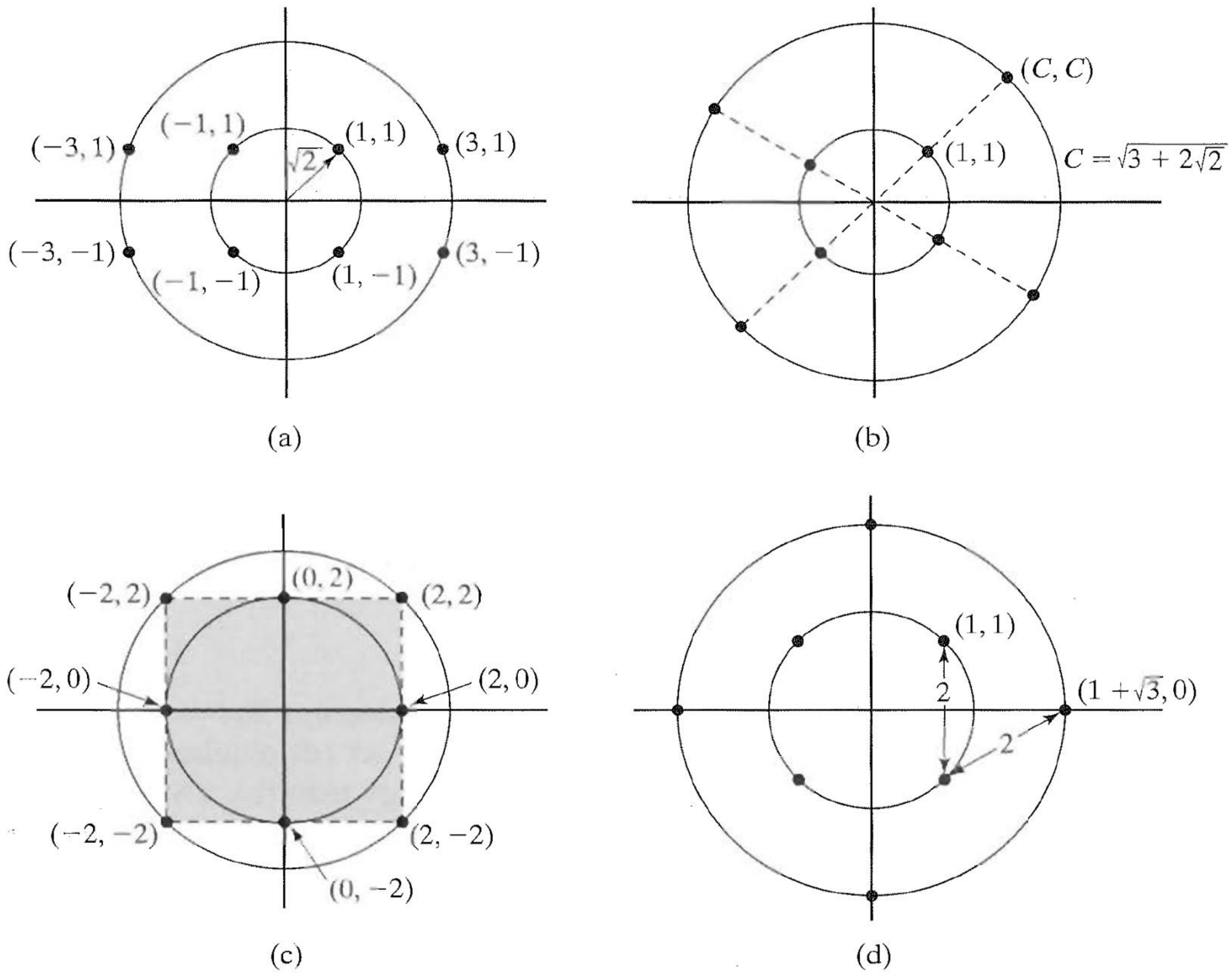

Figure 10.24 Four eight-point QAM signal constellations.

Probability of Error for QAM (4/8)

- Assuming that the constellation points are equally probable, the average transmitted signal energy is

$$\mathcal{E}_{av} = \mathcal{E}_s \frac{1}{M} \sum_{m=1}^{M} (A_{mc}^2 + A_{ms}^2),$$

- The two signal sets (a) and (c) contain signal points that have $\mathcal{E}_{av} = 6\mathcal{E}_s$. The signal set (b) requires an average transmitted signal $\mathcal{E}_{av} = 6.82\mathcal{E}_s$, and the signal set (d) requires $\mathcal{E}_{av} = 4.73\mathcal{E}_s$
- The signal constellation (d) is the best eight-point QAM constellation among the four cases because it requires the least energy for a given minimum distance between signal points

Probability of Error for QAM (5/8)

- For $M \geqq 16$, there are many more possibilities for selecting the QAM signal points in the two-dimensional space. An example 16-point QAM signal constellation is shown in Fig. 10.25. However, this is not the best 16-QAM for the AWGN channel

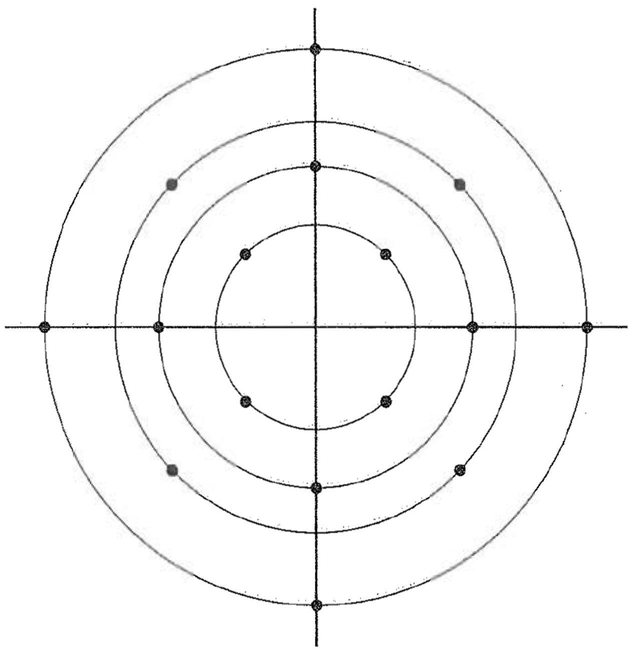

Figure 10.25 Circular 16-point QAM signal constellation.

Probability of Error for QAM (6/8)

- Rectangular QAM signal constellations have the distinct advantage of being easily generated as two PAM signals impressed on phase-quadrature carriers. They are easily demodulated as previously described
- Although they are not the best M-ary QAM signal constellations for $M \geqq 16$, the average transmitted energy required to achieve a given minimum distance is only slightly greater than the average energy required for the best M-ary QAM signal constellation. For these reasons, rectangular M-ary QAM signals are most frequently used in practice

Probability of Error for QAM (7/8)

- If we employ the optimum detector that bases its decisions on the optimum distance metrics, it can be shown that the symbol error probability is tightly upper-bounded as

$$P_M \leq 4Q\left(\sqrt{\frac{3k\,\mathcal{E}_{bav}}{(M-1)N_0}}\right) \tag{10.3.20}$$

  for $k \geqq 1$, where $\mathcal{E}_{bav}/N_0$ is the average SNR/bit. Note we have $\mathcal{E}_{bav} = \mathcal{E}_{av}/k$
- The probability of a symbol error is plotted in Fig. 10.26 as a function of the average SNR/bit

Probability of Error for QAM (8/8)

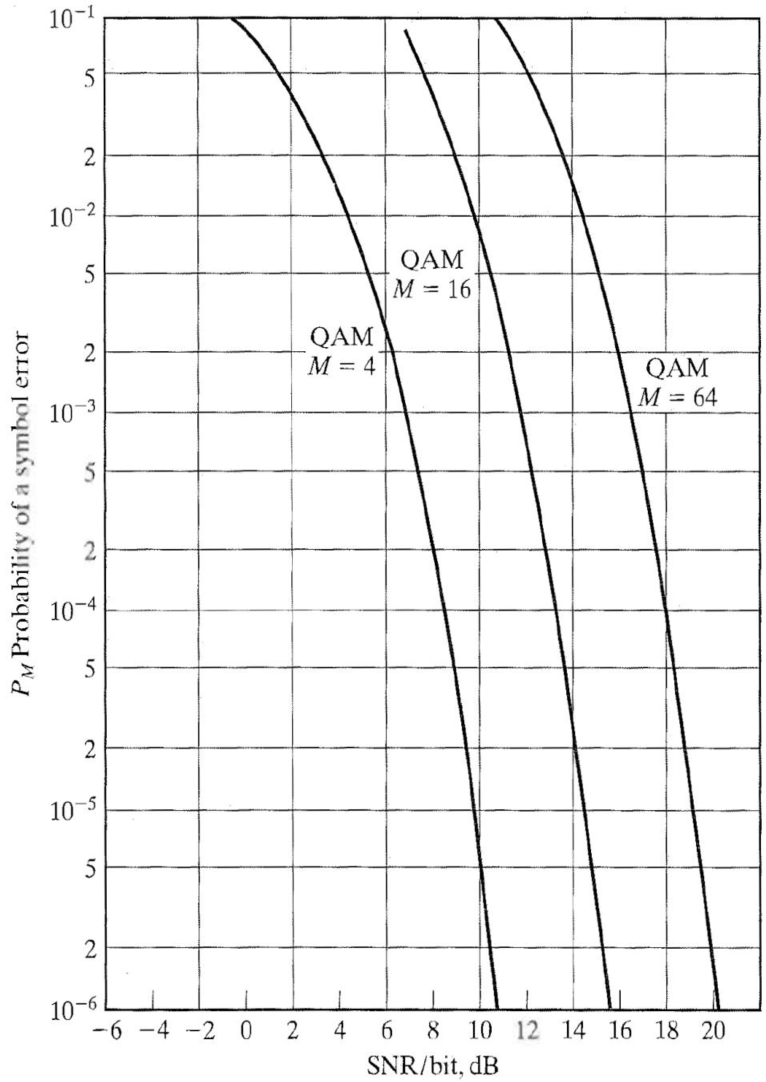

Figure 10.26 Probability of a symbol error for QAM.