

Chapter 4 Function Minimization Algorithms (II)

Quine-McCluskey for Multiple Output Problems (1/10)

- The Quine-McCluskey method can be expanded to include multiple output systems by adding a tag section to each product term. The tag indicates for which functions that term can be used.
- We indicate a term for a function with a - if the term is included in that function and a 0 if not. Terms can be combined if they have a common “-”.
- When combining terms (using the adjacency property), each tag is 0 if either term had a 0 and is “-” if both terms had a dash.

Quine-McCluskey for Multiple Output Problems (2/10)

- Consider finding all the prime implicants of functions defined as follows:

$$f(a,b,c) = \sum m(2,3,7)$$

$$g(a,b,c) = \sum m(4,3,7)$$

- We begin by grouping the terms based on the number of 1's
- Column I:

		tag	
A	010	-0	
B	100	0-	

C	011	-0	
D	101	0-	

E	111	-	

		<i>f</i>			
		<i>ab</i>	00	01	11
<i>c</i>	0	0	2 v	6	4
	1	1	3 v	7 v	5

		<i>g</i>			
		<i>ab</i>	00	01	11
<i>c</i>	0	0	2	6	4 v
	1	1	3	7 v	5 v

Quine-McCluskey for Multiple Output Problems (3/10)

- We now apply the adjacency property to each pair of terms in adjacent groups that have at least one “-” in common. The terms in Column II will be

$A + C = F =$	$0\ 1\ -\ -0$
$B + D = G =$	$1\ 0\ -\ 0\ -$

$C + E = H =$	$-1\ 1\ -0$
$D + E = J =$	$1\ -1\ 0\ -$

- When we continue to another column, terms are checked off if they are covered by other terms.

Quine-McCluskey for Multiple Output Problems (4/10)

- Term E is not checked off because it cannot be merged into terms F , G , H , and J

Table 4.7 Multiple output Quine-McCluskey method.

A 0 1 0 - 0 \checkmark	F 0 1 - - 0
B 1 0 0 0 - \checkmark	G 1 0 - 0 -
C 0 1 1 - 0 \checkmark	H - 1 1 - 0
D 1 0 1 0 - \checkmark	J 1 - 1 0 -
E 1 1 1 - -	

- There are 2 two-literal terms for each function and 1 three-literal term that can be shared

Quine-McCluskey for Multiple Output Problems (5/10)

- **Example 4.8.** Find all the primary implicants of the functions

$$f(a,b,c,d) = \sum m(2,3,4,6,9,11,12) + \sum d(0,1,14,15)$$

$$g(a,b,c,d) = \sum m(2,6,10,11,12) + \sum d(0,1,14,15).$$

- We begin by listing all of the minterms, with tags, including the don't cares, grouping terms by the number of 1's

		<i>f</i>			
		<i>ab</i> 00	01	11	10
<i>cd</i>	00	x	v	v	
	01	x			v
	11	v		x	v
	10	v	v	x	

		<i>g</i>			
		<i>ab</i> 00	01	11	10
<i>cd</i>	00	x		v	
	01	x			
	11			x	v
	10	v	v	x	v

Quine-McCluskey for Multiple Output Problems (6/10)

- Example 4.8. (Cont'd)**

Note that some sums, such as $AF+AN$ exist, have a tag of 00; they are not included.

A	0000 --√	AA	000- --	BA	00-- -0
	-----	AB	00-0 --	BB	0--0 -0
B	0001 --√	AC	0-00 -0√		-----
C	0010 --√		-----	BC	-0-1 -0
D	0100 -0√	AD	00-1 -0√	BD	--10 0-
	-----	AE	-001 -0√	BE	-1-0 -0
E	0011 -0√	AF	001- -0√		-----
F	0110 --√	AG	0-10 --	BF	1-1- 0-
G	1001 -0√	AH	-010 0-√		
H	1010 0-√	AI	01-0 -0√		
I	1100 --√	AJ	-100 -0√		
	-----		-----		
J	1011 --√	AK	-011 -0√		
K	1110 --√	AL	-110 --		
	-----	AM	10-1 -0√		
L	1111 --√	AN	101- 0-√		
		AO	1-10 0-√		
		AP	11-0 --		

		AQ	1-11 --		
		AR	111- --		

Term naming

Quine-McCluskey for Multiple Output Problems (7/10)

- **Example 4.8. (Cont'd)**
- The terms that can be shared are $a'b'c'$, $a'b'd'$, $a'cd'$, bcd' , abd' , acd , and abc .
- The prime implicants of f are $a'b'$, $a'd'$, $b'd$, and bd' . The prime implicants of g are cd' and ac .

Quine-McCluskey for Multiple Output Problems (8/10)

- **Example 4.9.** Find all the primary implicants of the functions:

$$f(x,y,z) = \sum m(0,2,5,6,7)$$

$$g(x,y,z) = \sum m(2,3,5,6,7)$$

$$h(x,y,z) = \sum m(0,2,3,4,5)$$

- The tag now has three bits, but otherwise the process is as before.

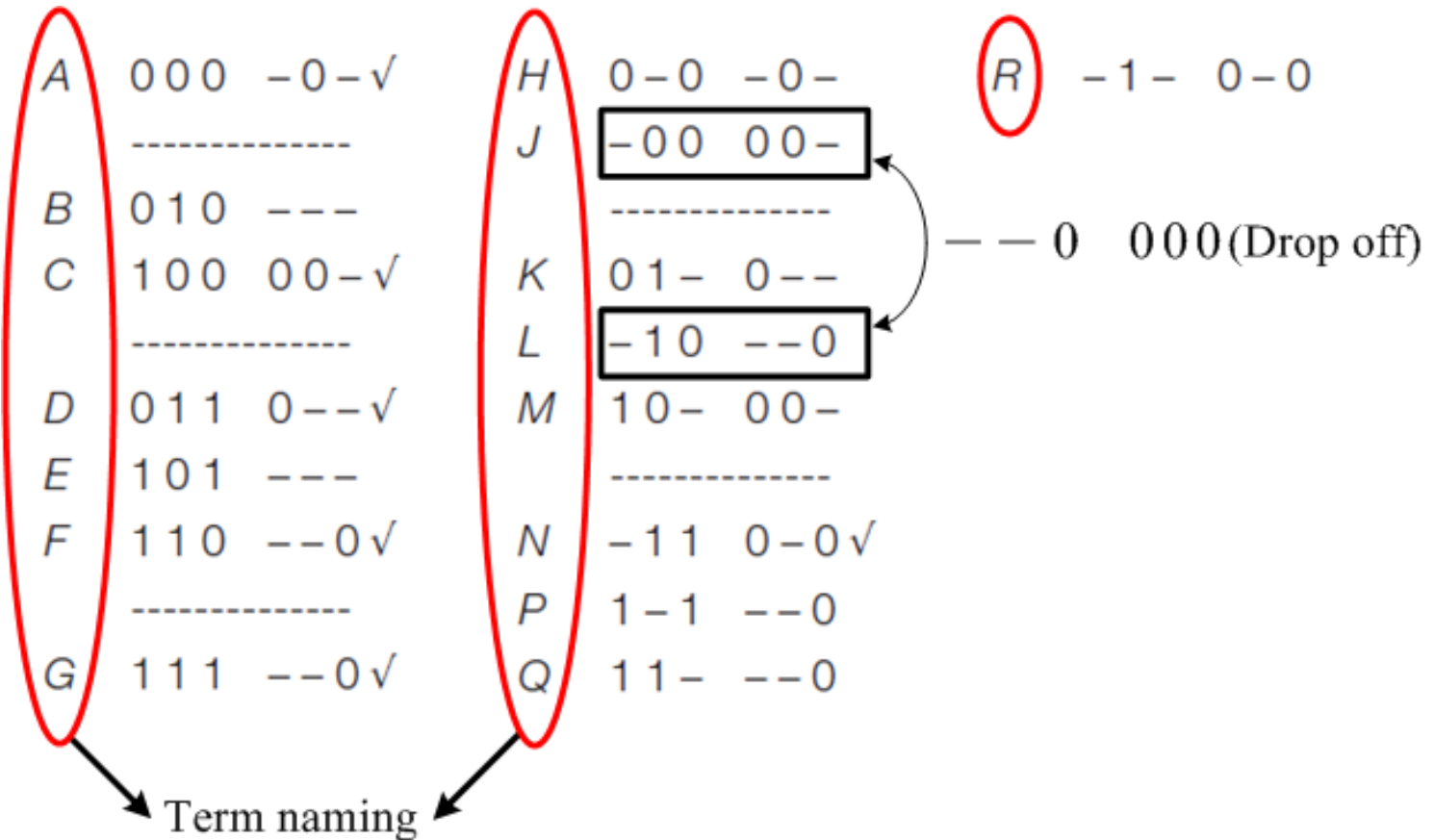
		<i>f</i>			
		<i>xy</i> 00	01	11	10
<i>z</i>	0	0 _v	2 _v	6 _v	4
	1	1	3	7 _v	5 _v

		<i>g</i>			
		<i>xy</i> 00	01	11	10
<i>z</i>	0	0	2 _v	6 _v	4
	1	1	3 _v	7 _v	5 _v

		<i>h</i>			
		<i>xy</i> 00	01	11	10
<i>z</i>	0	0 _v	2 _v	6	4 _v
	1	1	3 _v	7	5 _v

Quine-McCluskey for Multiple Output Problems (9/10)

- **Example 4.9. (Cont'd)**



Quine-McCluskey for Multiple Output Problems (10/10)

- **Example 4.9. (Cont'd)**
- The terms that can be used for all three functions (can be shared) are $x'yz'$ and $xy'z$
- f and g can use yz' , xz , and xy
- f and h can use $x'z'$
- g and h can use $x'y$
- h can use $y'z'$ and xy'
- g can use y

Iterated Consensus for Multiple Output Problems (1/6)

- The iterated consensus algorithm needs only minor modifications to produce all of the terms that may be used for sum of products expressions for multiple output problems
- We will find that all terms that were shared between two functions were indeed prime implicants of the product of those two functions, and terms that were shared among three functions were prime implicants of the product of the three functions
- We can either start with minterms or a cover of each function plus a cover of all possible products of functions

Iterated Consensus for Multiple Output Problems (2/6)

- We illustrate the iterated consensus algorithm using the functions

$$f(a,b,c) = \Sigma_m(2,3,7)$$

$$g(a,b,c) = \Sigma_m(4,5,7)$$

- The initial list becomes

2	<i>a'</i>	<i>b</i>	<i>c'</i>	0	1	0	–	0
3	<i>a'</i>	<i>b</i>	<i>c</i>	0	1	1	–	0
4	<i>a</i>	<i>b'</i>	<i>c'</i>	1	0	0	0	–
5	<i>a</i>	<i>b'</i>	<i>c</i>	1	0	1	0	–
7	<i>a</i>	<i>b</i>	<i>c</i>	1	1	1	–	–

Tag for *g*

⇓

– 0

– 0

0 –

0 –

– –

⇑

Tag for *f*

Iterated Consensus for Multiple Output Problems (3/6)

- We now proceed as before, taking the consensus of each pair of terms, adding new terms and deleting terms included in others

Iterated Consensus for Multiple Output Problems (4/6)

Table 4.8 Iterated consensus for multiple output functions.

$a' bc'$	A	0	1	0	$-$	0	}	Initial terms
$a' bc$	B	0	1	1	$-$	0		
$ab' c'$	C	1	0	0	0	$-$		
$ab' c$	D	1	0	1	0	$-$		
abc	E	1	1	1	$-$	$-$		
$a' b$	F	0	1	$-$	$-$	0		$B \zeta A \geq B, A$ $B \zeta A = a' b$
ab'	G	1	0	$-$	0	$-$		$D \zeta C \geq D, C$ $D \zeta C = ab'$
bc	H	$-$	1	1	$-$	0		$F \zeta E$ $F \zeta E = bc$
ac	J	1	$-$	1	0	$-$		$G \zeta E$ $(G \zeta F \text{ undefined}) G \zeta E = ac$

$H \zeta G$ zero tag; $H \zeta F, H \zeta E$ undefined
 $J \zeta H, J \zeta F$ zero tag; $J \zeta G, J \zeta E$ undefined

Zero tag: no output uses it

- The term that can be shared is abc ; $a'b$ and bc are prime implicants of f ; ab' and ac are prime implicants of g

Iterated Consensus for Multiple Output Problems (5/6)

- **Example 4.11.** Find all the prime implicants of the functions

$$f(x,y,z) = \sum m(0,2,5,6,7)$$

$$g(x,y,z) = \sum m(2,3,5,6,7)$$

$$h(x,y,z) = \sum m(0,2,3,4,5)$$

- We start by listing all of the minterms used for any of the functions, and then perform the iterated consensus algorithm to find all of the prime implicants.

Iterated Consensus for Multiple Output Problems (6/6)

- Example 4.11. (Cont'd)**

A	0 0 0	- 0 -	H	0 - 0	- 0 -	$B \not\subseteq A \geq A$
B	0 1 0	- - -	J	0 1 -	0 - -	$C \not\subseteq B \geq C$
C	0 1 1	0 - -	K	1 0 -	0 0 -	$E \not\subseteq D \geq D$
D	1 0 0	0 0 -	L	- 1 0	- - 0	$F \not\subseteq B \geq F$
E	1 0 1	- - -	M	1 - 1	- - 0	$G \not\subseteq E \geq G$
F	1 1 0	- - 0	N	- 0 0	0 0 -	$K \not\subseteq H$
G	1 1 1	- - 0	P	1 1 -	- - 0	$M \not\subseteq L$
			Q	- 1 1	0 - 0	$M \not\subseteq J$
			R	- 1 -	0 - 0	$Q \not\subseteq L \geq Q$