Chapter 8 Digital Modulation in an Additive White Gaussian Noise Baseband Channel (II)

Binary Pulse Position Modulation (1/7)

- In binary PPM, we employ two signal waveforms, $s_1(t)$ and $s_2(t)$, which are shown in Fig. 8.10. $s_1(t)$ and $s_2(t)$ are orthogonal, *i.e.*,

$$\int_0^{T_b} s_1(t)s_2(t)dt = 0$$

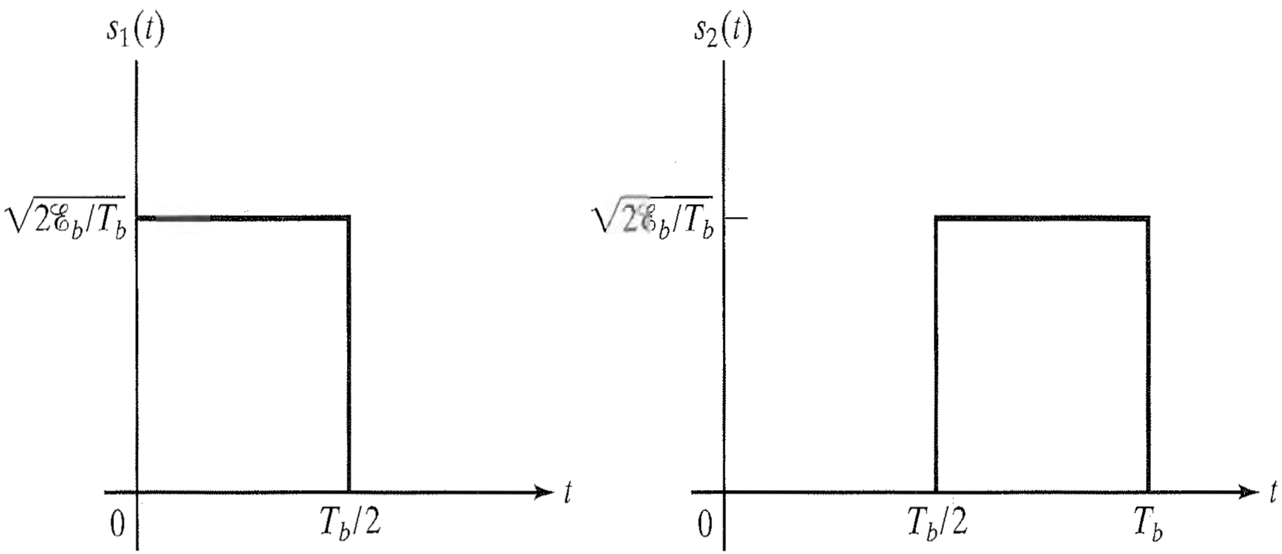


Figure 8.10 Signal pulses in binary PPM (orthogonal signals).

Binary Pulse Position Modulation (2/7)

- The two signal waveforms have identical energies, *i.e.*,

$$\mathcal{E}_b = \int_0^{T_b} s_1^{\,2}(t)dt = \int_0^{T_b} s_2^{\,2}(t)dt$$

- In order to represent these two waveforms geometrically as vectors, we need two nonoverlapping (orthogonal) basis functions, and each must be normalized to unit energy. These two basis functions, $\psi_1(t)$ and $\psi_2(t)$, are shown in Fig. 8.11

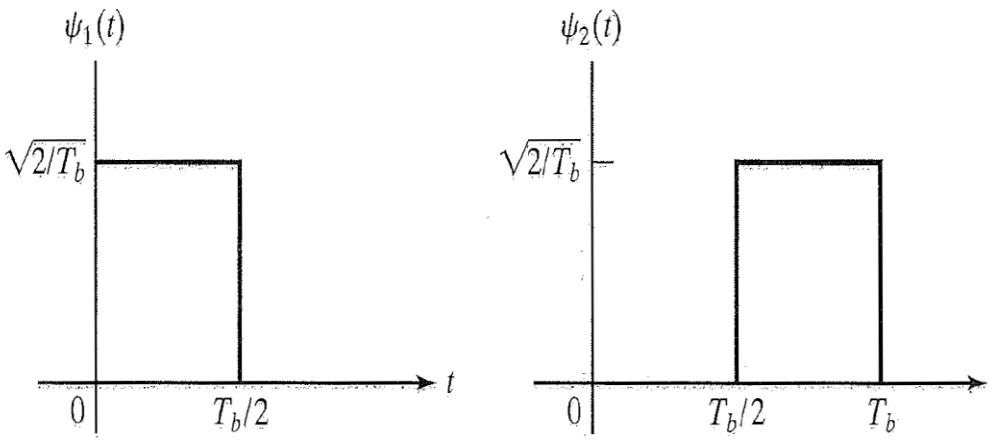


Figure 8.11 Two orthogonal basis functions for binary PPM signals.

Binary Pulse Position Modulation (3/7)

- The signal waveforms $s_1(t)$ and $s_2(t)$ may be expressed as

$$s_1(t) = s_{11}\psi_1(t) + s_{12}\psi_2(t)$$

$$s_2(t) = s_{21}\psi_1(t) + s_{22}\psi_2(t)$$

where we can easily observe that

$$s_{11} = \int_0^{T_b} s_1(t)\psi_1(t)dt = \sqrt{\mathcal{E}_b}$$

$$s_{12} = \int_0^{T_b} s_1(t)\psi_2(t)dt = 0$$

$$s_{21} = \int_0^{T_b} s_2(t)\psi_1(t)dt = 0$$

$$s_{22} = \int_0^{T_b} s_2(t)\psi_2(t)dt = \sqrt{\mathcal{E}_b}$$

Binary Pulse Position Modulation (4/7)

- The two signal waveforms are represented as two-dimensional vectors $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ when

$$\boldsymbol{s}_1=(s_{11},0)=(\sqrt{\mathcal{E}_b},0)$$
$$\boldsymbol{s}_2=(0,s_{22})=(0,\sqrt{\mathcal{E}_b})$$

- The two signals are perpendicular; hence, they are orthogonal , *i.e.*, their dot product is equal to zero

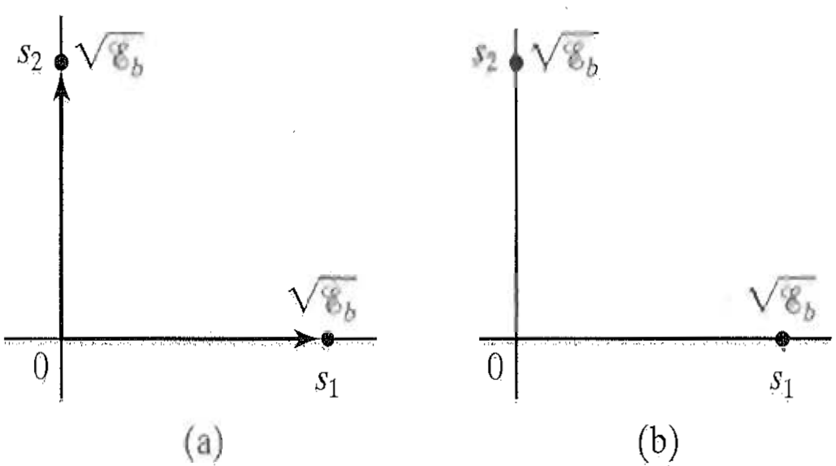


Figure 8.12 Geometric representation of binary orthogonal (PPM) signal waveforms.

Binary Pulse Position Modulation (5/7)

- **Example 8.2.2.** Consider the two orthogonal signal waveforms shown in Fig. 8.13. Show that these two signal waveforms have a similar geometric representation as the two PPM pulses shown in Fig. 8. 12.

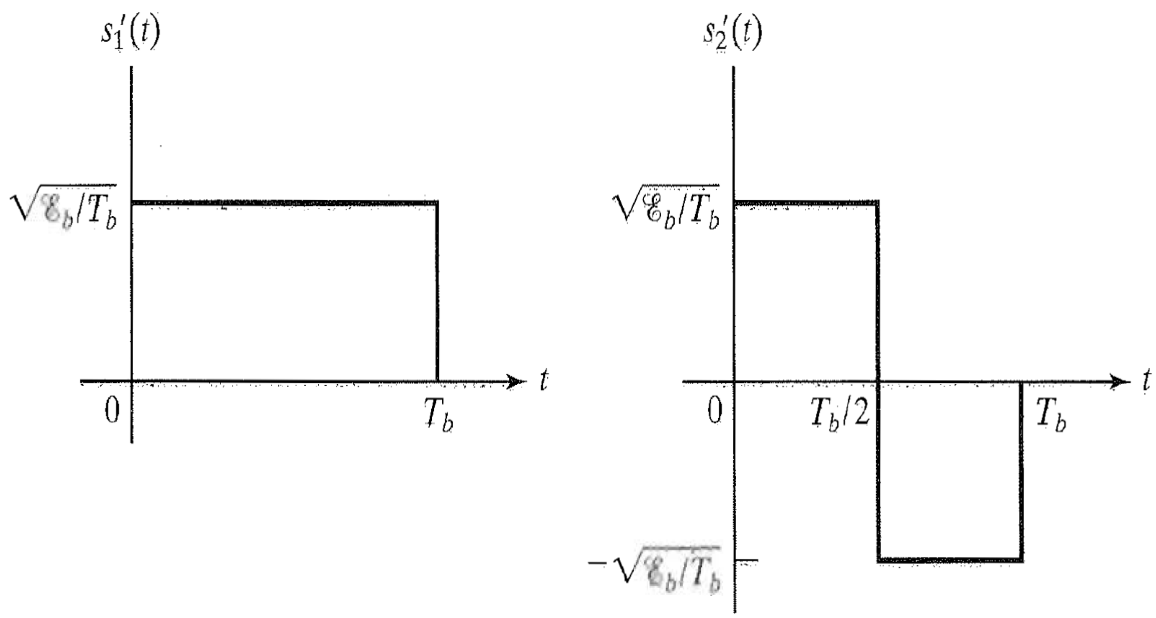


Figure 8.13 Two orthogonal signal waveforms.

Binary Pulse Position Modulation (6/7)

- **Example 8.2.2. (Cont'd)** By using the orthonormal basis waveforms $\psi_1(t)$ and $\psi_2(t)$ in Fig. 8.11, the signal waveform $s_1'(t)$ and $s_2'(t)$ are expressed as

$$s_1'(t) = s_{11}'\psi_1(t) + s_{12}'\psi_2(t)$$

and

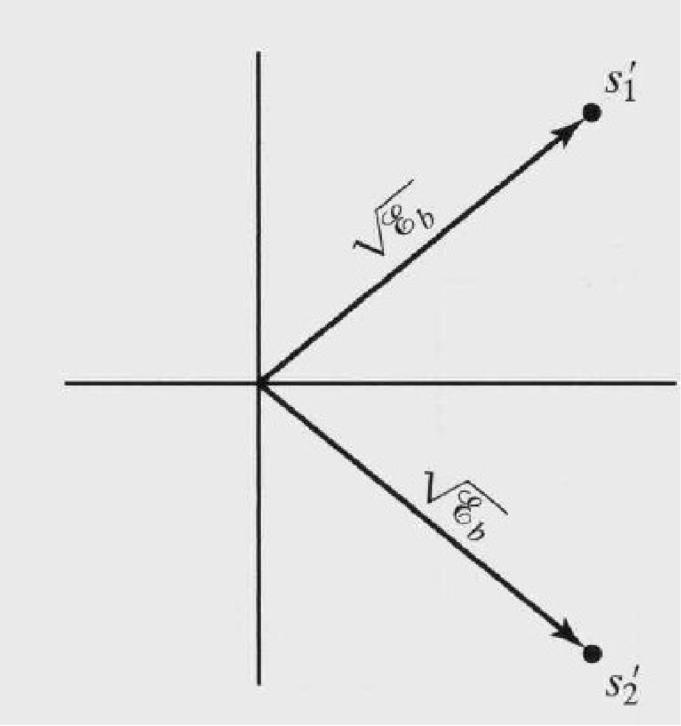
$$s_2'(t) = s_{21}'\psi_1(t) + s_{22}'\psi_2(t),$$

where

$$s_{11}' = \sqrt{\mathcal{E}_b/2}$$
$$s_{12}' = \sqrt{\mathcal{E}_b/2}$$
$$s_{21}' = \sqrt{\mathcal{E}_b/2}$$
$$s_{22}' = -\sqrt{\mathcal{E}_b/2}$$

Binary Pulse Position Modulation (7/7)

- **Example 8.2.2. (Cont'd)** The vectors s_1' and s_2' are shown in the figure. We observe that s_1' and s_2' are perpendicular (orthogonal vectors) and are simply a phase-rotated version of the orthogonal vectors shown in Fig. 8.12(a)



Optimum Receiver for Binary Modulation Signals in Additive White Gaussian Noise (1/4)

- We begin by describing the channel that corrupts the transmitted signal by the addition of noise
- The communication channel is assumed to corrupt the transmitted signal by the addition of white Gaussian noise

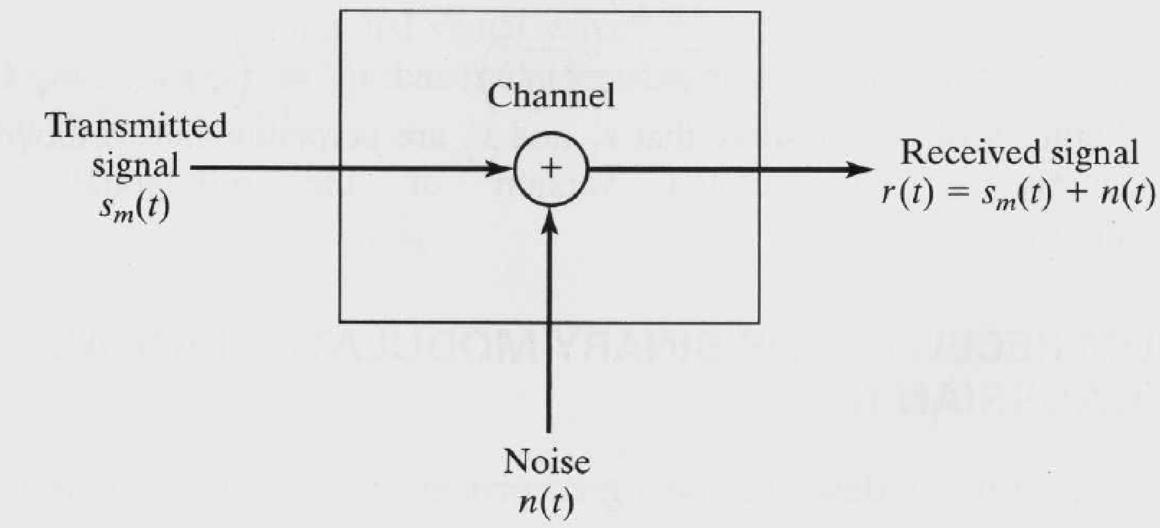


Figure 8.15 Model for the received signal passed through an AWGN channel.

Optimum Receiver for Binary Modulation Signals in Additive White Gaussian Noise (2/4)

- The received signal in a signal interval of duration T_b may be expressed as

$$r(t)=s_m(t)+n(t) \quad m=1,\ 2$$

  where $n(t)$ denotes the sample function of the additive white Gaussian noise (AWGN) process with the power spectral density $S_n(f)=\frac{N_0}{2}$ W/Hz

- Based on the observation of $r(t)$ over the signal interval, we wish to design a receiver that is optimum in the sense that it minimizes the probability of making an error. We focus on the processing of the received signal $r(t)$ in the interval $0 \leqq t \leqq T_b$

Optimum Receiver for Binary Modulation Signals in Additive White Gaussian Noise (3/4)

- We subdivide the receiver into two parts, the signal demodulator and the detector, as shown in Fig. 8.16
- The signal demodulator's function is to convert the received signal waveform $r(t)$ into a vector y, whose dimension is equal to the dimension of the transmitted signal waveforms
- The detector's function is to decide which of the two possible signal waveforms was transmitted based on y

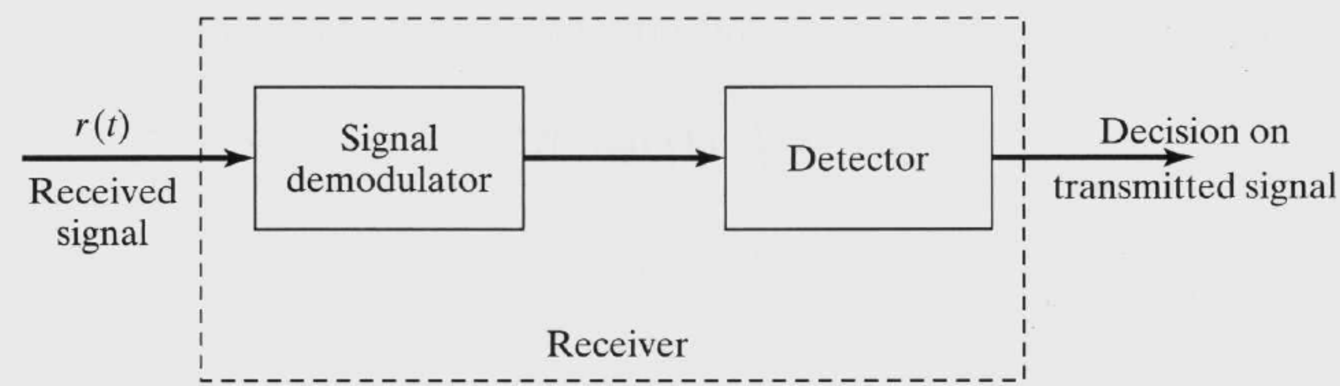


Figure 8.16 Receiver for digitally modulated signals.

Optimum Receiver for Binary Modulation Signals in Additive White Gaussian Noise (4/4)

- There are two realizations of the signal demodulator. The first is based on the use of signal correlators. The second is based on the use of matched filters
- The optimum detector that follows the signal demodulator is designed to minimize the probability of error

Correlation-Type Demodulator (1/6)

- We consider the processing of the received signal by a correlation-type demodulator for binary antipodal signal (binary PAM) and binary orthogonal signals (binary PPM)
- We first consider binary antipodal signals

$$s_m(t) = s_m \psi(t), \qquad m = 1, 2,$$

where $\psi(t)$ is the unit energy rectangular pulse shown in Fig. 8.6 and $s_1 = \sqrt{\mathcal{E}_b}$, $s_2 = -\sqrt{\mathcal{E}_b}$

- The received signal is

$$r(t) = s_m \psi(t) + n(t),\ 0 \leqq t \leqq T_b,\ m = 1, 2$$

Correlation-Type Demodulator (2/6)

- In a correlation-type demodulator, the received signal $r(t)$ is multiplied by the signal waveform $\psi(t)$ and the product is integrated over the interval $0 \leqq t \leqq T_b$. We say that $r(t)$ is cross-correlated with $\psi(t)$
- This cross-correlation operation produces the output

$$\begin{aligned} y(t) &= \int_0^t r(\tau)\psi(\tau)d\tau \\ &= \int_0^t [s_m\psi(\tau)+n(\tau)]\psi(\tau)d\tau \\ &= s_m\int_0^t \psi^2(\tau)d\tau + \int_0^t n(\tau)\psi(\tau)d\tau. \end{aligned} \qquad (8.3.4)$$

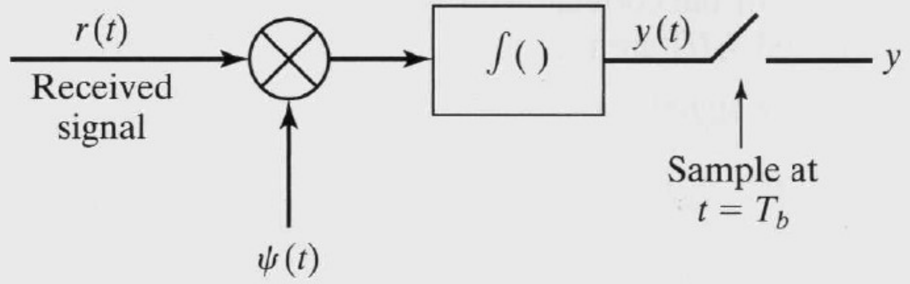


Figure 8.17 Cross-correlator for binary antipodal signals.

Correlation-Type Demodulator (3/6)

- We sample the output of the correlator at $t = T_b$. Thus,

$$y(T_b) = s_m + n,$$

where

$$n = \int_0^{T_b} \psi(\tau) n(\tau) d\tau.$$

- Since $n(t)$ is a sample function of a white Gaussian noise process, the noise term n is a Gaussian random variable with zero mean and with variance

$$\begin{aligned}
\sigma_n^2 = E(n^2) &= \int_0^{T_b} \int_0^{T_b} E[n(t)n(\tau)] \psi(t)\psi(\tau) dt d\tau \\
&= \int_0^{T_b} \int_0^{T_b} \frac{N_0}{2} \delta(t-\tau) \psi(t)\psi(\tau) dt d\tau \\
&= \frac{N_0}{2} \int_0^{T_b} \psi^2(t) dt \\
&= \frac{N_0}{2}
\end{aligned}$$

Correlation-Type Demodulator (4/6)

- For a given signal transmission (given s_m), the output of the correlator (y=$y(T_b)$) is a Gaussian random variable with mean s_m and variance $N_0/2$, *i.e.*,

$$f(y\,|\,s_m) = \frac{1}{\sqrt{\pi N_0}} e^{-(y-s_m)^2/N_0}, \quad m=1,\,2$$

- These two conditional probability density functions are illustrated in Fig. 8.18. This correlator output is fed to the detector, which decides whether the transmitted bit is a zero or a one

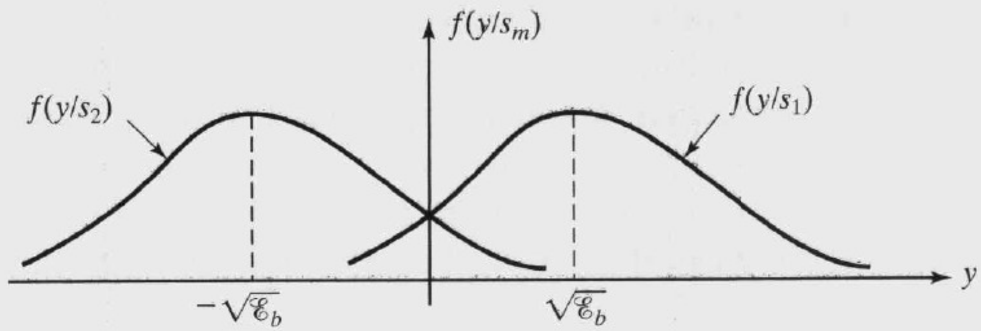


Figure 8.18 The conditional probability density functions of the correlator output for binary antipodal signaling.

Correlation-Type Demodulator (5/6)

- **Example 8.3.1.** Sketch the noise-free output of the correlator for the rectangular pulse $\psi(t)$, as shown in Fig. 8.6, when $s_1(t)$ and $s_2(t)$ are transmitted.
- From (8.3.4) and $n(t)=0$, the signal waveform at the output of the correlator is

$$y(t) = s_m \int_0^t \psi^2(\tau)d\tau.$$

The graphs of $y(t)$ for $s_1=\sqrt{\mathcal{E}_b}$ and $s_2=-\sqrt{\mathcal{E}_b}$ are shown in Fig. 8.19

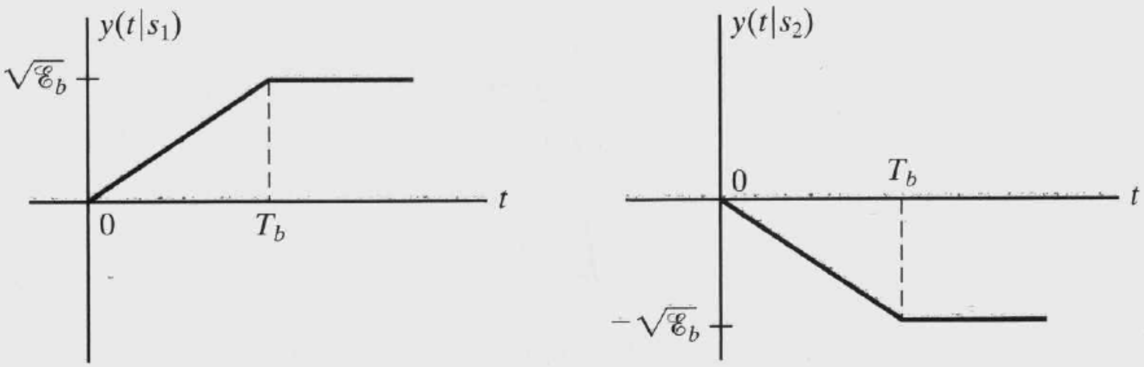


Figure 8.19 Noise-free cross-correlator outputs when $s_1(t)$ and $s_2(t)$ are transmitted.

Correlation-Type Demodulator (6/6)

- **Example 8.3.1. (Cont'd)** We observe that the maximum signal at the output of the correlator occurs at $t=T_b$. We also observe that the correlator must be reset to zero at the end of each bit interval T_b, so that it can be used in the demodulation of the received signal in the next signal interval. Such an integrator is called an *integrate-and-dump filter*