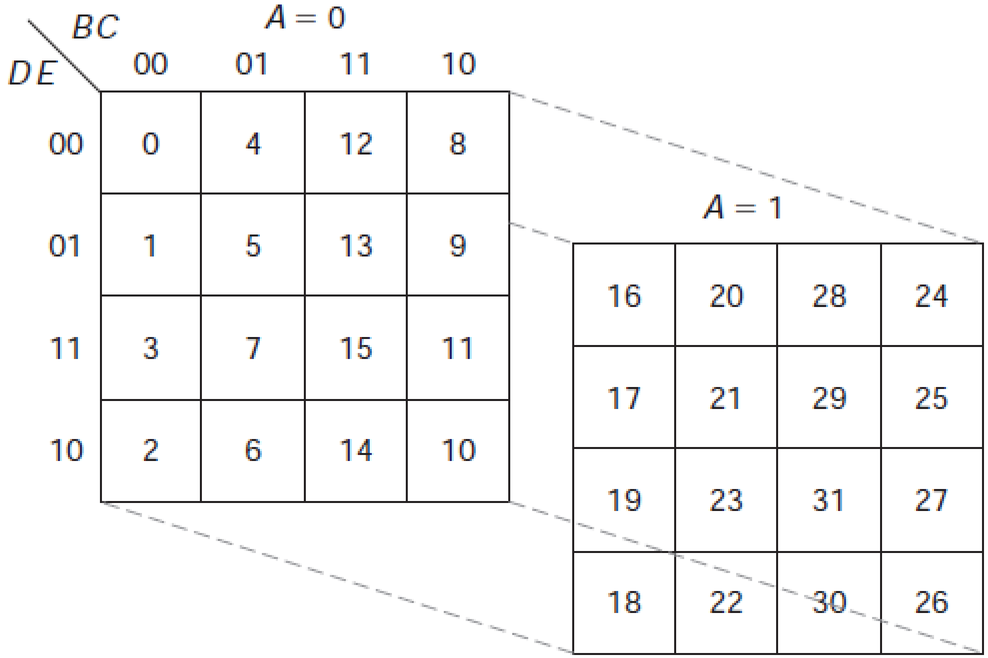
Chapter 3 The Karnaugh Map (III)

Five- and Six-Variable Maps (1/17)

- A five-variable map consists of $2^5 = 32$ squares. We prefer to look at it as two layers of 16 squares each. Each square in the bottom layer corresponds to the minterm numbered 16 more than the square above it.

Map 3.16 A five-variable map.



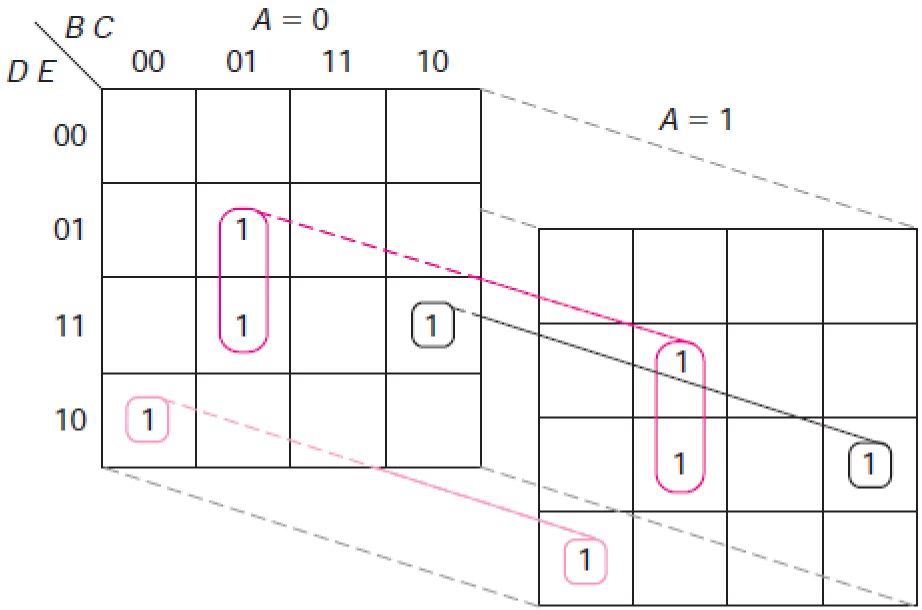
Five- and Six-Variable Maps (2/17)

- **Example 3.27.** Circle the following minterms on the Karnaugh map

$$m_2 + m_{18} = A'B'C'DE' + AB'C'DE' = B'C'DE'$$

$$m_{11} + m_{27} = A'BC'DE + ABC'DE = BC'DE$$

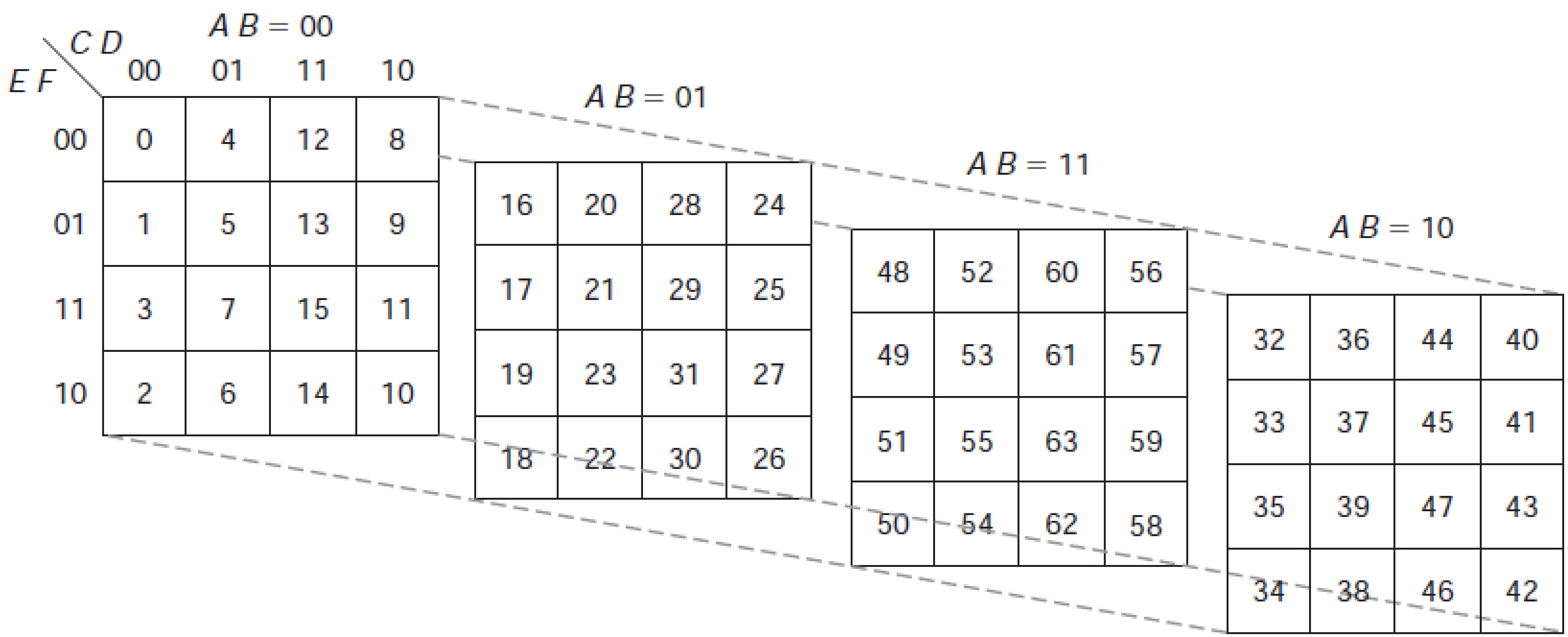
$$m_5 + m_7 + m_{21} + m_{23} = B'CE$$



Five- and Six-Variable Maps (3/17)

- Six-variable maps are drawn as four layers of 16-square maps, where the first two variables determine the layer and the other variables specify the square within the layer

Map 3.17 A six-variable map.



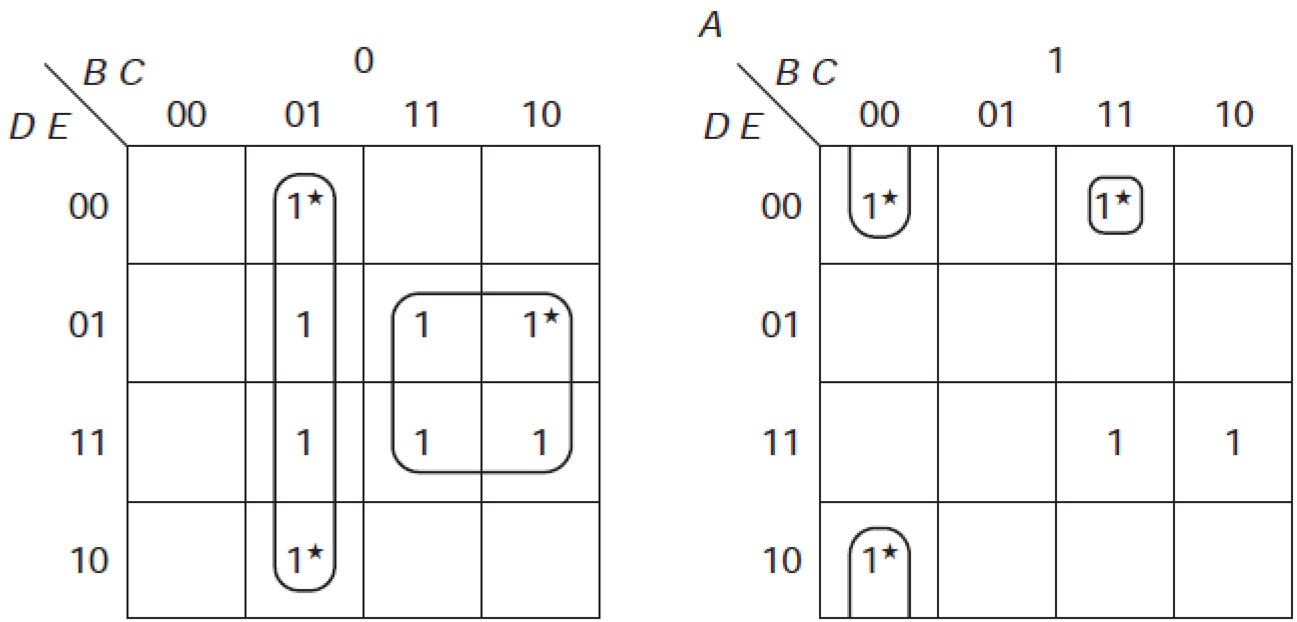
Five- and Six-Variable Maps (4/17)

- To simplify a five-variable map, we first look for the essential prime implicants. A good staring point is to find 1's on one layer for which there is a 0 in the corresponding square on the adjoining layer.
- Prime implicants that cover that 1 are contained completely on that layer. Thus, we have a four-variable map problem.

Five- and Six-Variable Maps (5/17)

- Consider the following map.

Map 3.19 Essential prime implicants on one layer.



- So far, we have

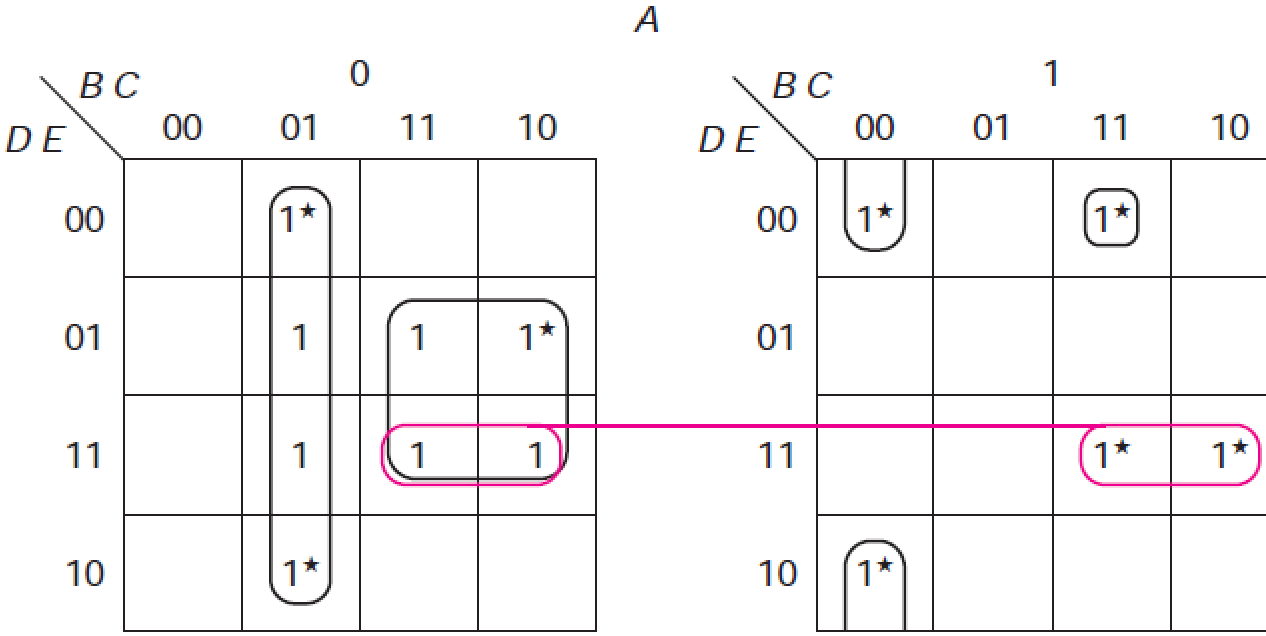
$$F = A'B'C + A'BE + AB'C'E' + ABCD'E' + \cdots$$

Five- and Six-Variable Maps (6/17)

- The two 1's remaining uncovered do have counterparts on the other layer. The only prime implicant that covers them is *BDE*. The complete solution is

$$F = A'B'C + A'BE + AB'C'E' + ABCD'E' + BDE$$

Map 3.20 A prime implicant covering 1's on both layers.



Five- and Six-Variable Maps (7/17)

- **Example 3.28.** Obtain a minimum SOP form for the function

$$G(A, B, C, D, E) = \Sigma m(1, 3, 8, 9, 11, 12, 14, 17, 19, 20, 22, 24, 25, 27)$$

A = 0

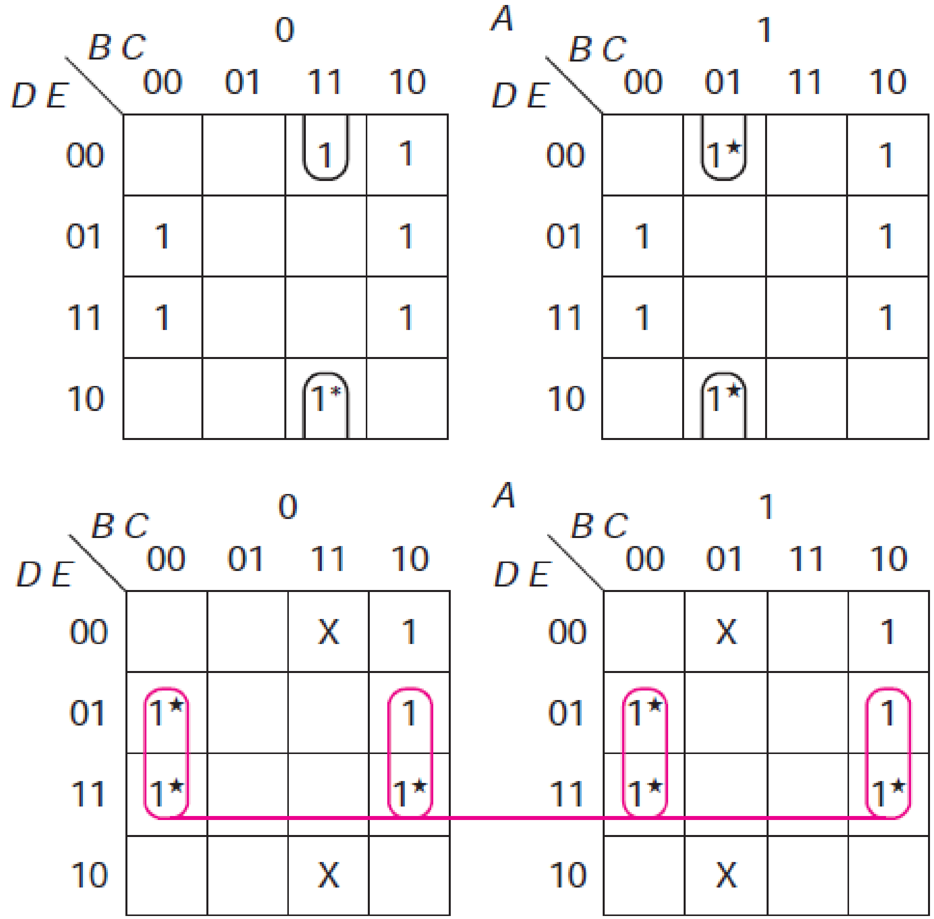
DE \ BC	00	01	11	10
00			1	1
01	1			1
11	1			1
10			1	

A = 1

DE \ BC	00	01	11	10
00		1		1
01	1			1
11	1			1
10		1		

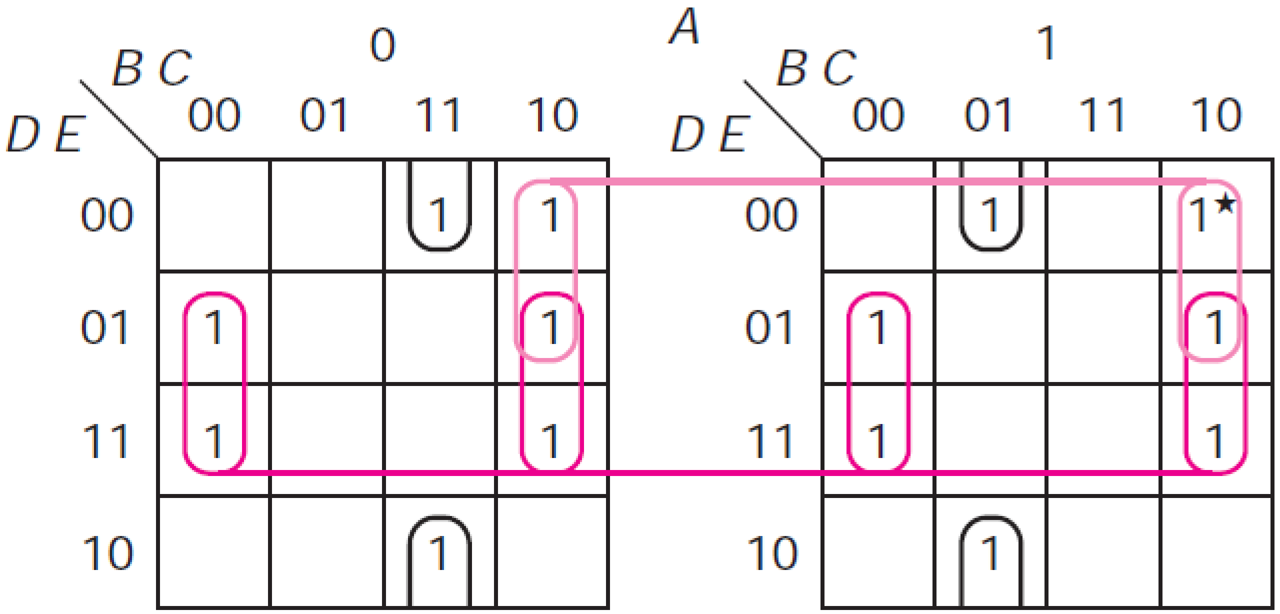
Five- and Six-Variable Maps (8/17)

- **Example 3.28 (Cont'd)** The essential prime implicant on the second map are shown as don't cares



Five- and Six-Variable Maps (9/17)

- **Example 3.28 (Cont'd)**



- The solution is

$$G = A'BCE' + AB'CE' + C'E + BC'D'$$

Five- and Six-Variable Maps (10/17)

- **Example 3.29.** Obtain the function from the maps.

A = 0

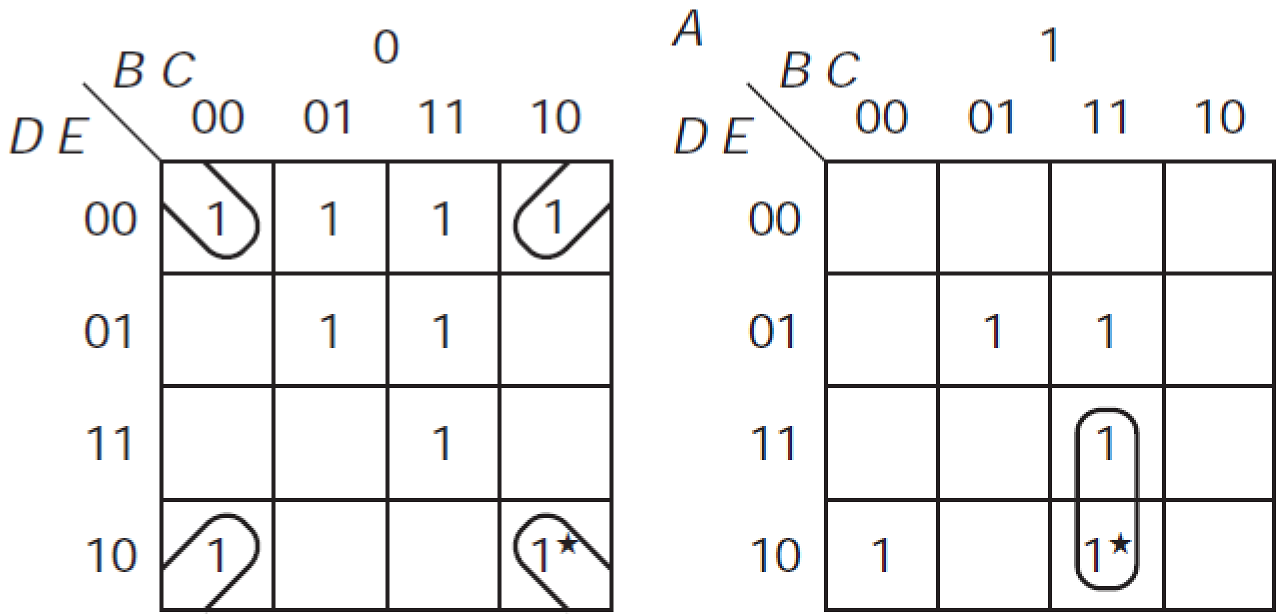
DE \ BC	00	01	11	10
00	1	1	1	1
01		1	1	
11			1	
10	1			1

A = 1

DE \ BC	00	01	11	10
00				
01		1	1	
11			1	
10	1		1	

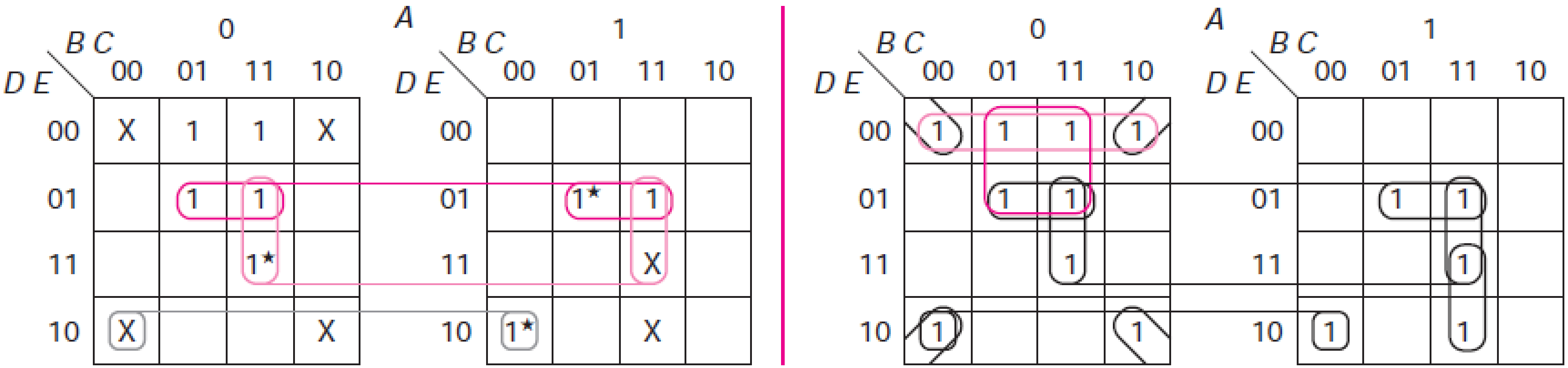
Five- and Six-Variable Maps (11/17)

- **Example 3.29 (Cont'd)** We have



Five- and Six-Variable Maps (12/17)

- **Example 3.29 (Cont'd)**



- The two solutions are

$$F = A'C'E' + ABCD + CD'E + BCE + B'C'DE' + A'CD'$$

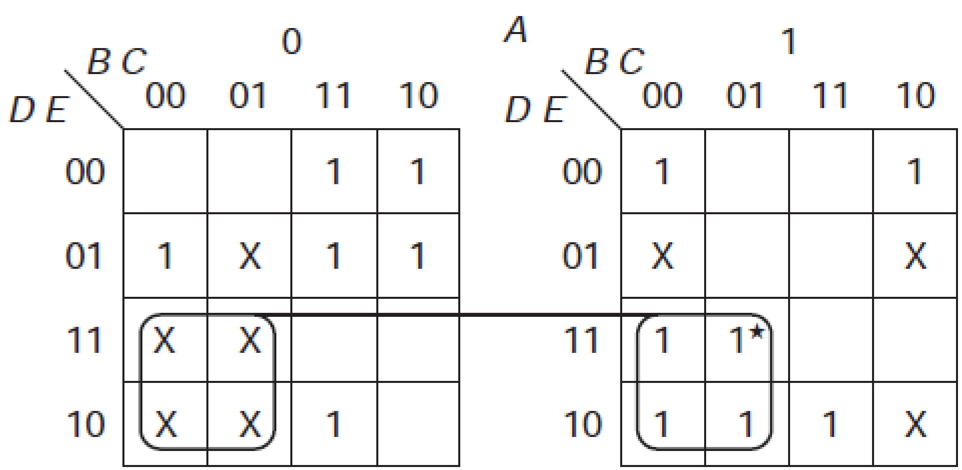
$$F = A'C'E' + ABCD + CD'E + BCE + B'C'DE' + A'D'E'$$

Five- and Six-Variable Maps (13/17)

- **Example 3.30.** Consider the function

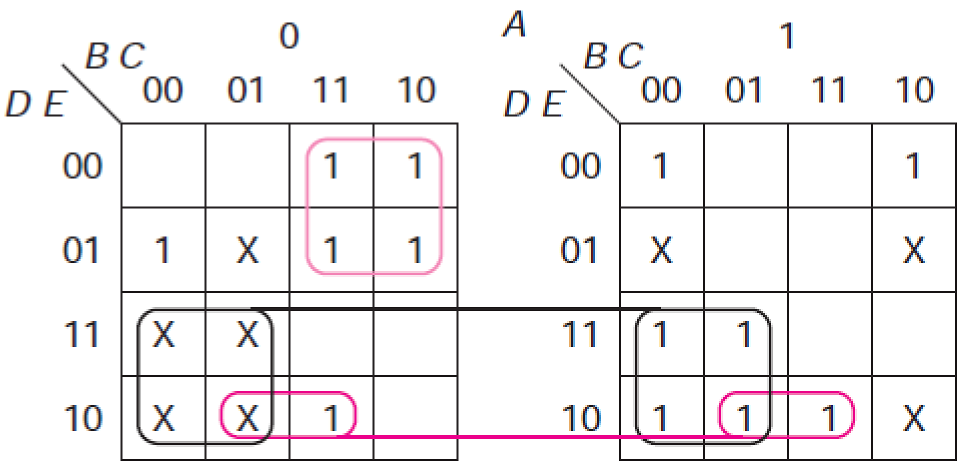
$$H(A, B, C, D, E) = \Sigma m(1, 8, 9, 12, 13, 14, 16, 18, 19, 22, 23, 24, 30) + \Sigma d(2, 3, 5, 6, 7, 17, 25, 26)$$

- A map of *H* is shown below with the only essential prime implicant, *B'D* (a group of eight, including four 1's and four don't cares), circled

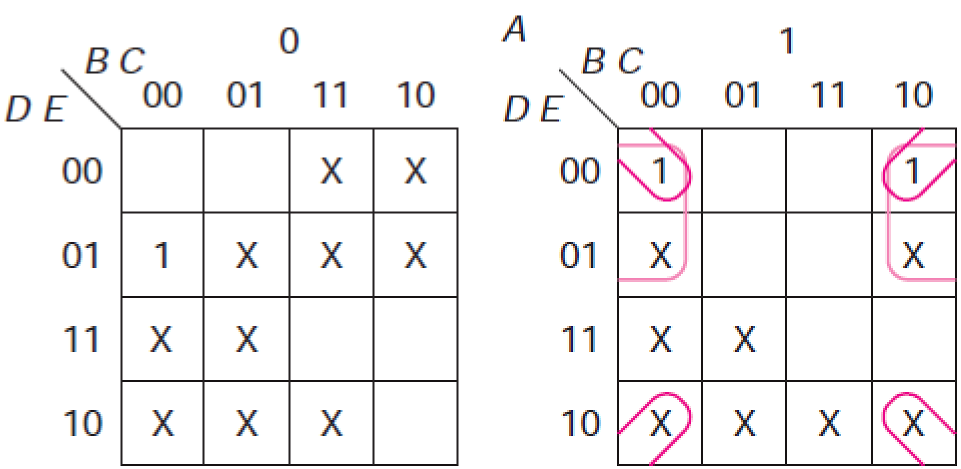


Five- and Six-Variable Maps (14/17)

- **Example 3.30 (Cont'd)**



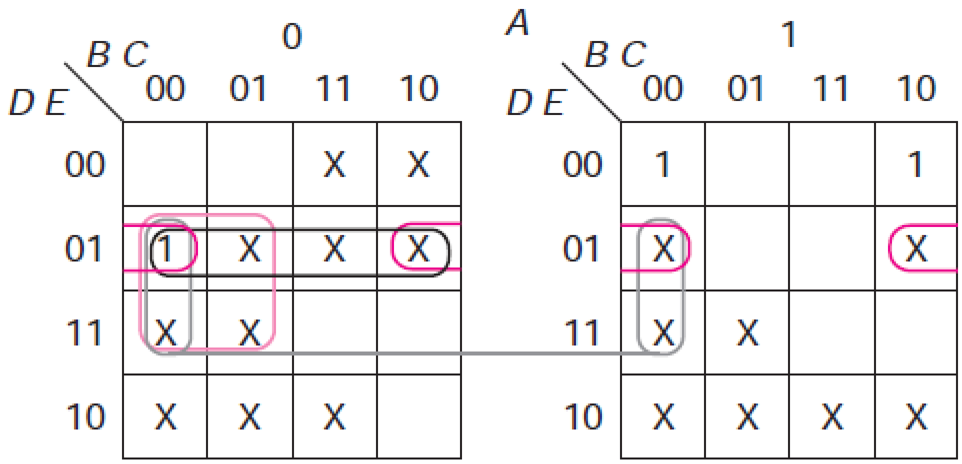
$\Rightarrow B'D + CDE' + A'BD'$



$\Rightarrow \begin{Bmatrix} AC'E' \\ AC'D' \end{Bmatrix}$

Five- and Six-Variable Maps (15/17)

- **Example 3.30 (Cont'd)**



$$\Rightarrow \begin{Bmatrix} A'D'E \\ A'B'E \\ B'C'E \\ C'D'E \end{Bmatrix}$$

- The eight solutions are

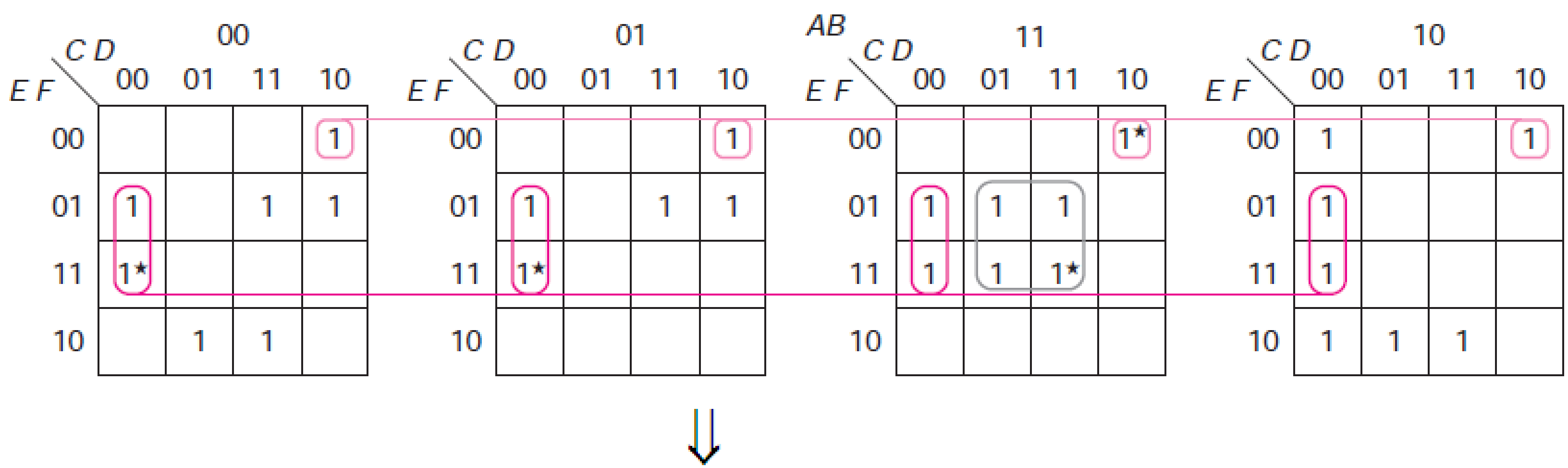
$$H = B'D + CDE' + A'BD' + \begin{Bmatrix} AC'E' \\ AC'D' \end{Bmatrix} + \begin{Bmatrix} A'D'E \\ A'B'E \\ B'C'E \\ C'D'E \end{Bmatrix}$$

Five- and Six-Variable Maps (16/17)

- **Example 3.31.** Consider the function

$$G(A, B, C, D, E, F) = \Sigma m(1, 3, 6, 8, 9, 13, 14, 17, 19, 24, 25, 29, 32, 33, 34, 35, 38, 40, 46, 49, 51, 53, 55, 56, 61, 63)$$

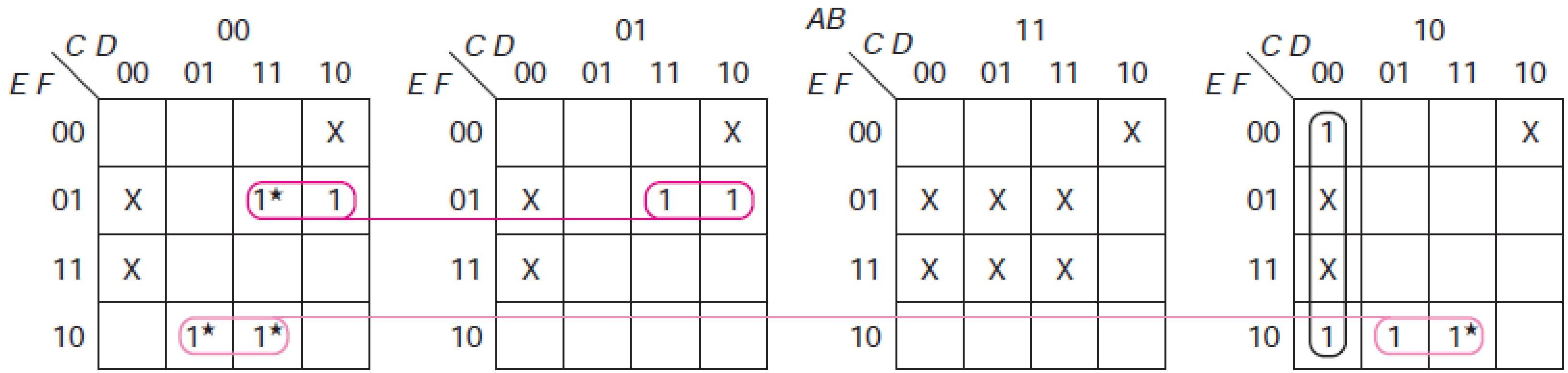
- There are more than three essential prime implicants.



$$ABDF + CD'E'F' + C'D'F$$

Five- and Six-Variable Maps (17/17)

- **Example 3.31 (Cont'd)** The next map shows 1's covered by the first three prime implicants as don't cares.



$$\Downarrow$$

$$A'CE'F + B'DEF' + AB'C'D'$$

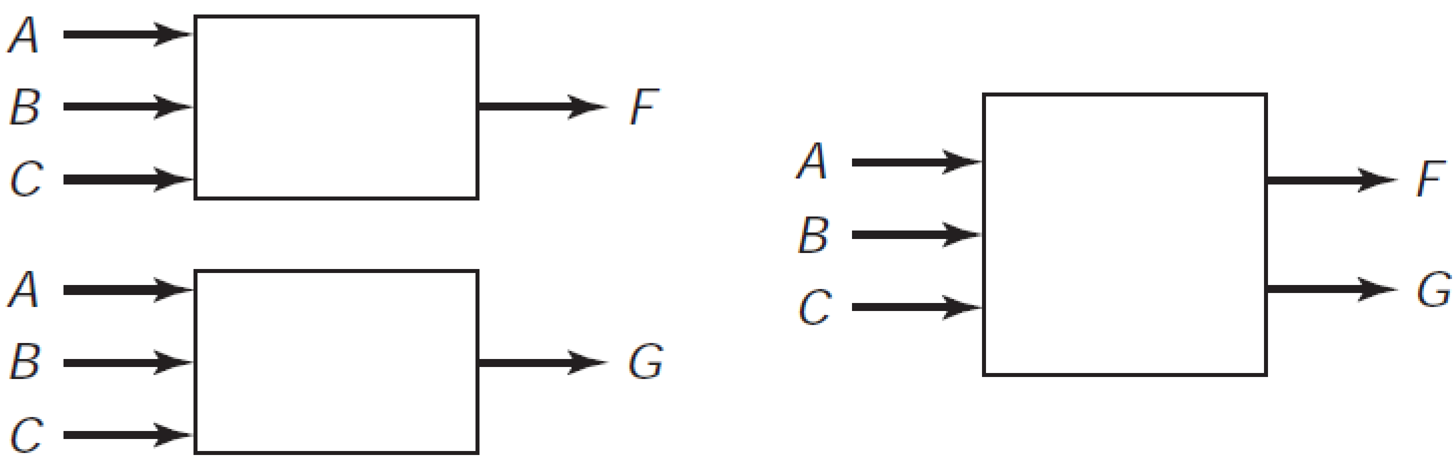
- Remember that the top and bottom layers are adjacent. The minimum expression is

$$G = ABDF + CD'E'F' + C'D'F + A'CE'F + B'DEF' + AB'C'D'$$

Multiple Output Problems (1/14)

- If, for example, we had a problem with three inputs, *A*, *B*, and *C* and two outputs, *F* and *G*, we could treat this as two separate problems. However, if we treated this as a single system with three inputs and two outputs, we may be able to economize by sharing gates

Figure 3.1 Implementation of two functions.



Two Separate Systems

One System

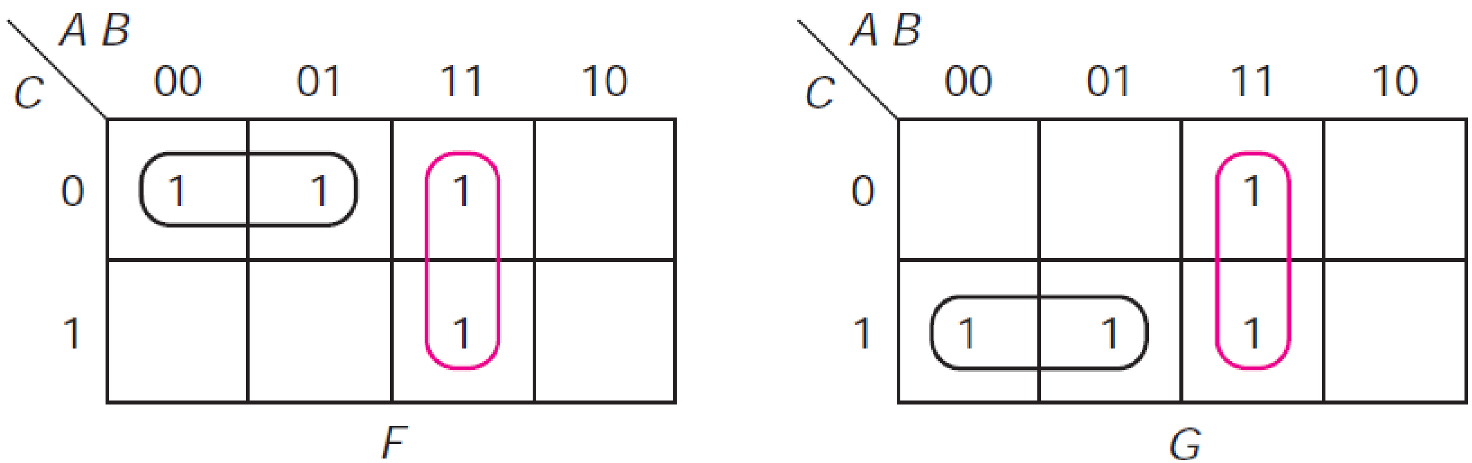
Multiple Output Problems (2/14)

- **Example 3.32.** Consider two functions

$$F(A, B, C) = \Sigma m(0, 2, 6, 7) \qquad G(A, B, C) = \Sigma m(1, 3, 6, 7)$$

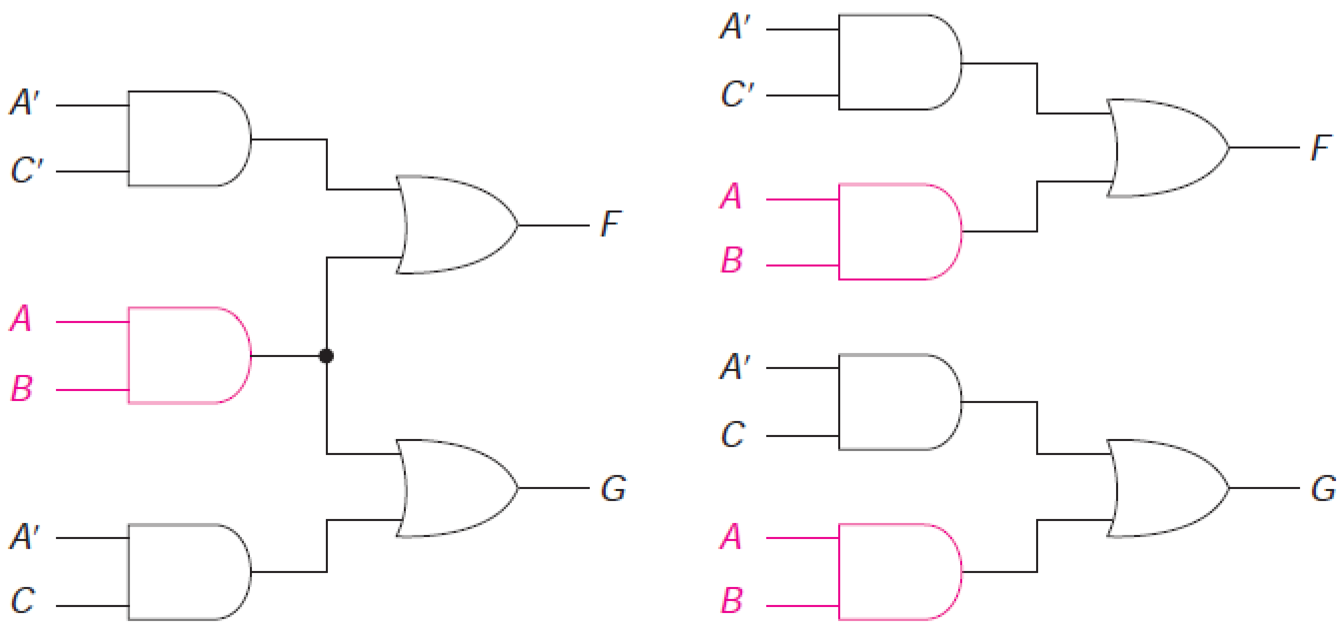
- If we map each of these and solve them separately, we obtain

$$F = A'C' + AB \qquad G = A'C + AB$$



Multiple Output Problems (3/14)

- **Example 3.32. (Cont'd)** The same term (*AB*) is circled on both and can be shared. We will use as the definition for *minimum* a circuit containing the minimum number of gates, and among those with the same number of gates, the minimum number of gate inputs.

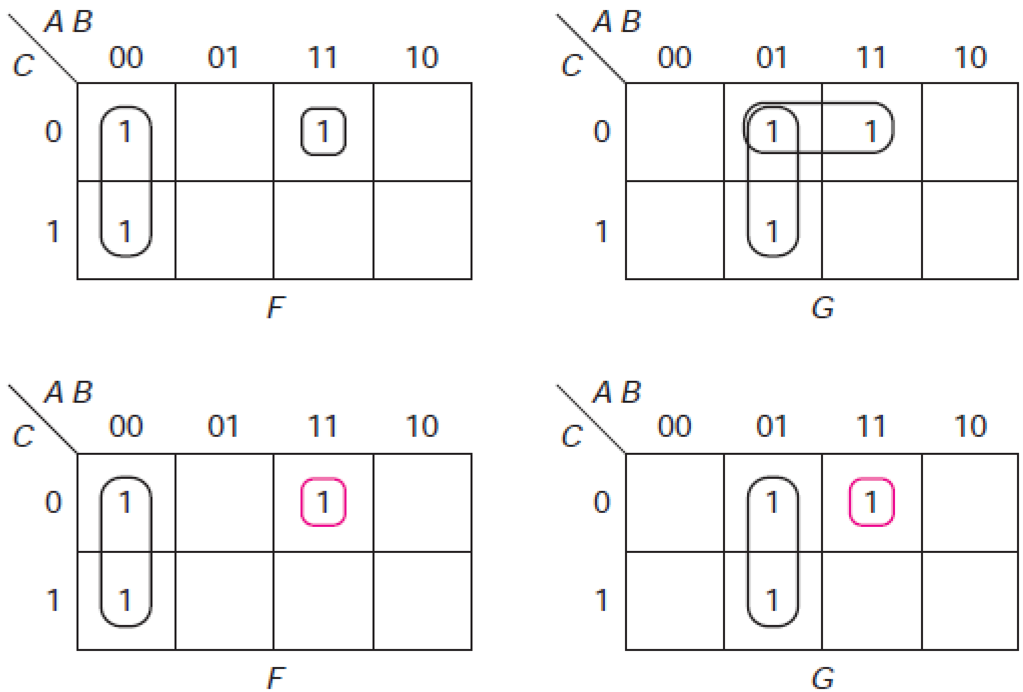


Multiple Output Problems (4/14)

- **Example 3.33.** Consider the two functions

$$F(A, B, C) = \Sigma m(0, 1, 6) \qquad G(A, B, C) = \Sigma m(2, 3, 6)$$

- Even when the two functions do not have a common prime implicants, some parts can still be shared.



Multiple Output Problems (5/14)

- **Example 3.33 (Cont'd).** In the top maps, we consider each function separately and obtained

  $$F = A'B' + ABC' \qquad G = A'B + BC'$$

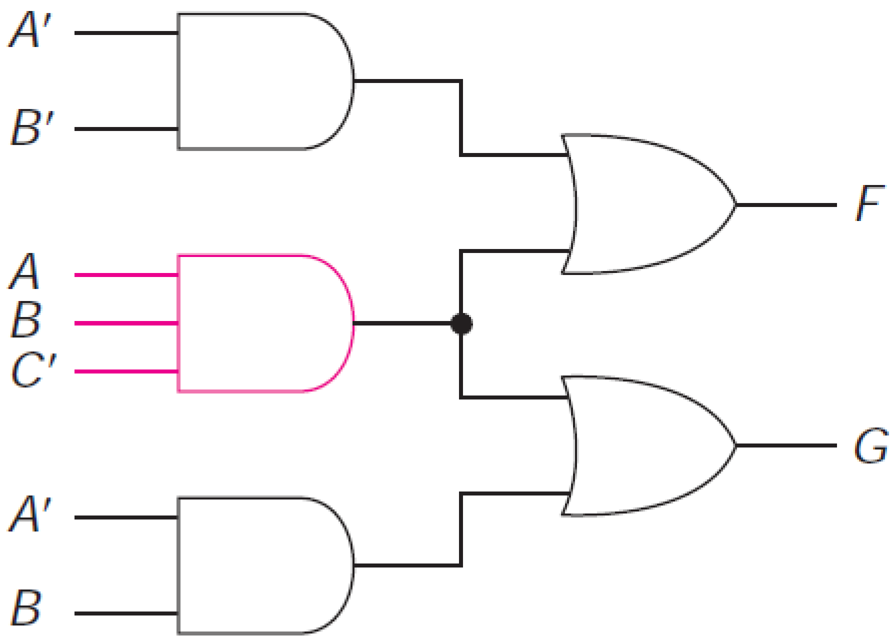
  This solution requires six gates (four ANDs and two ORs) with 13 inputs

- However, as we share the term *ABC'* and obtain

  $$F = A'B' + ABC' \qquad G = A'B + ABC'$$

Multiple Output Problems (6/14)

- **Example 3.33 (Cont'd).** As can be seen from the circuit, this only requires five gates with 11 inputs



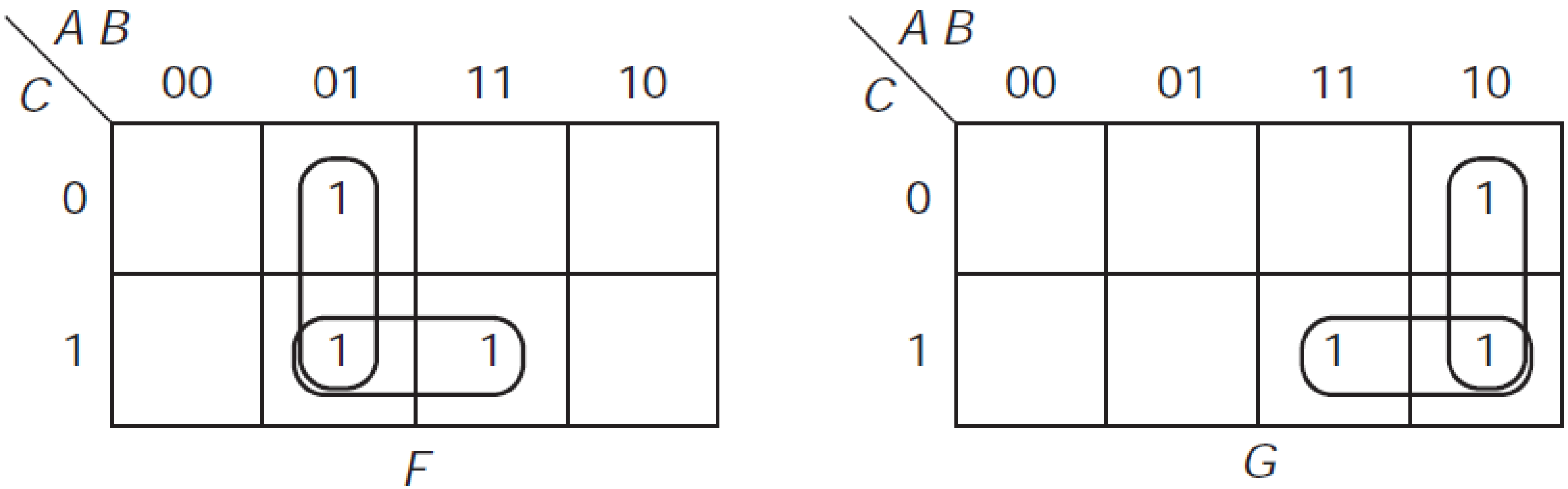
Multiple Output Problems (7/14)

- **Example 3.34.** Consider the two functions

$$F(A, B, C) = \Sigma m(2, 3, 7) \qquad G(A, B, C) = \Sigma m(4, 5, 7)$$

- Using essential prime implicants of each function, we obtain

$$F = A'B + BC \qquad G = AB' + AC$$

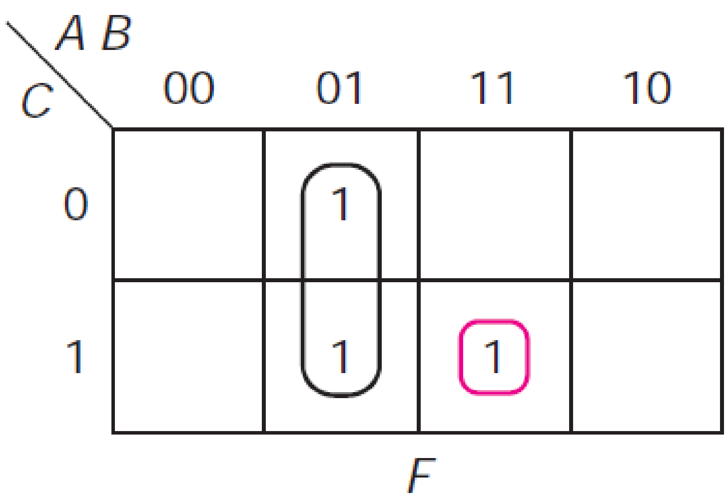


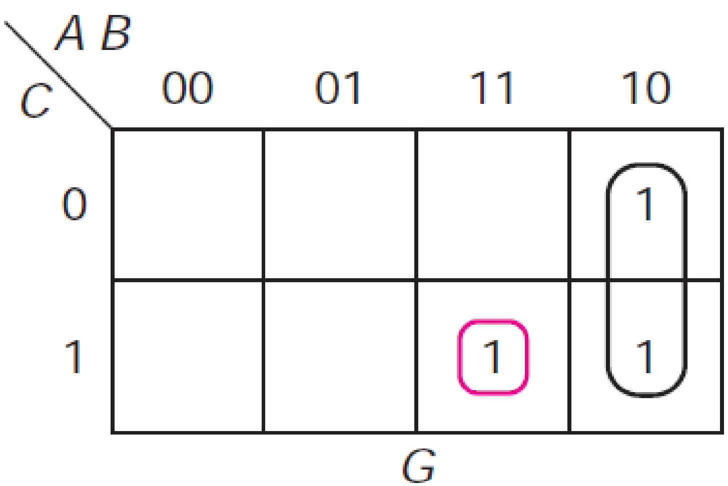
Multiple Output Problems (8/14)

- **Example 3.34 (Cont'd)**

  We can share the term *ABC*, even though it is not a prime implicant of either function, and get a solution that requires only five gates:

$$F = A'B + ABC \qquad G = AB' + ABC$$





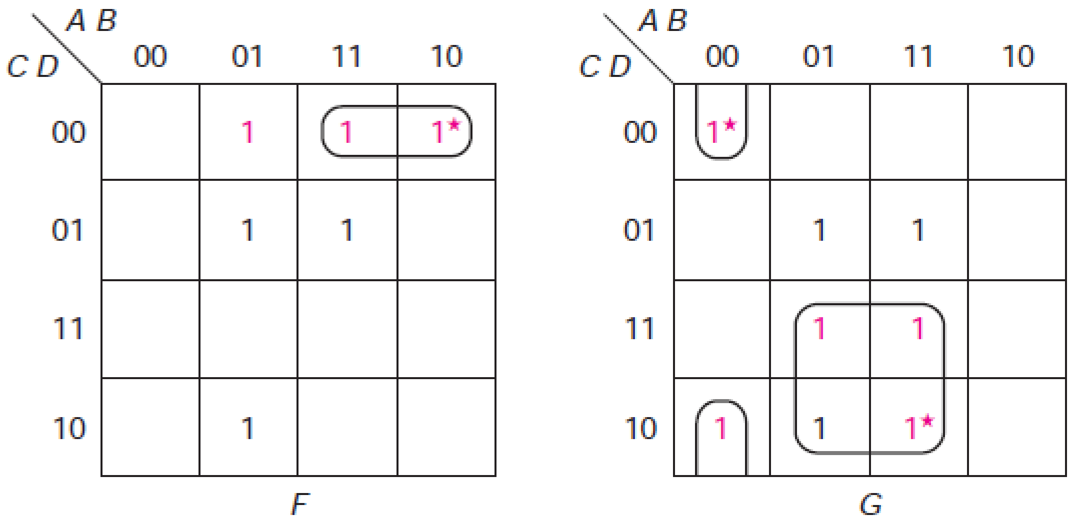
Multiple Output Problems (9/14)

- **Example 3.35.** Consider the two functions

$$F(A, B, C, D) = \Sigma m(4, 5, 6, 8, 12, 13)$$

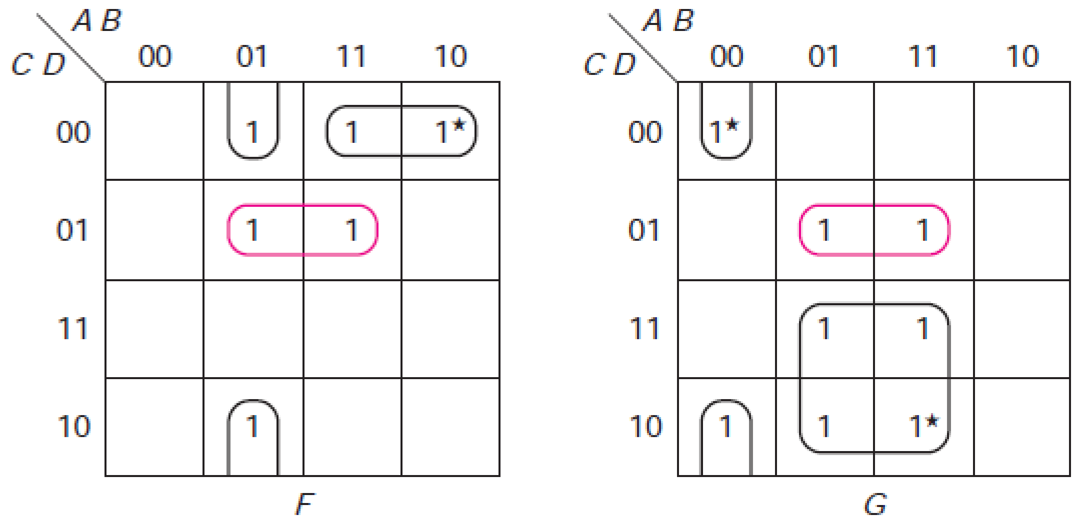
$$G(A, B, C, D) = \Sigma m(0, 2, 5, 6, 7, 13, 14, 15)$$

- The maps of these functions are shown below. In them, we have shown in pink the 1's that are included in one function and not the other



Multiple Output Problems (10/14)

- **Example 3.35 (Cont'd).** We have



leaving

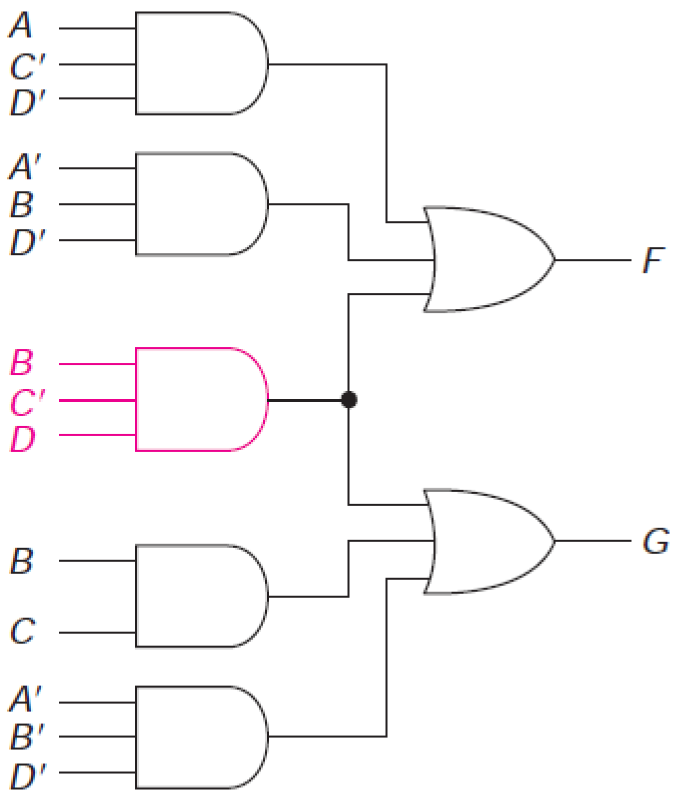
$$F = AC'D' + A'BD' + BC'D$$

$$G = A'B'D' + BC + BC'D$$

- With sharing, a total of 7 gates and 20 gate inputs is required. In contrast, that of no sharing requires a total of eight gates

Multiple Output Problems (11/14)

- **Example 3.35 (Cont'd).** The shared version of the circuit is shown below



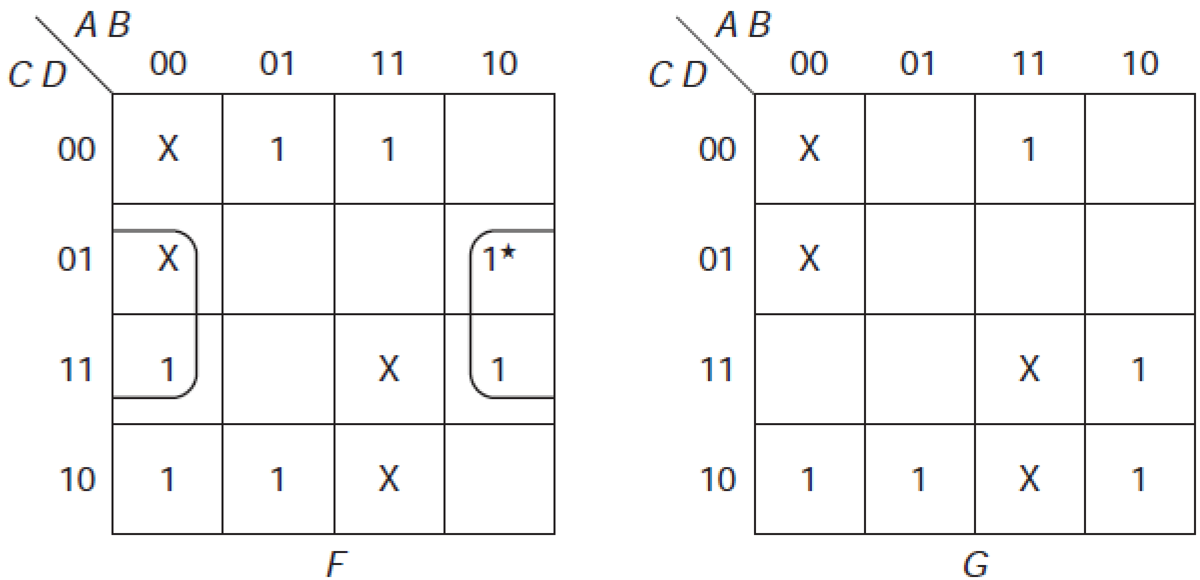
Multiple Output Problems (12/14)

- **Example 3.40.** Consider an example with don’t cares:

$$F(A, B, C, D) = \Sigma m(2, 3, 4, 6, 9, 11, 12) + \Sigma d(0, 1, 14, 15)$$

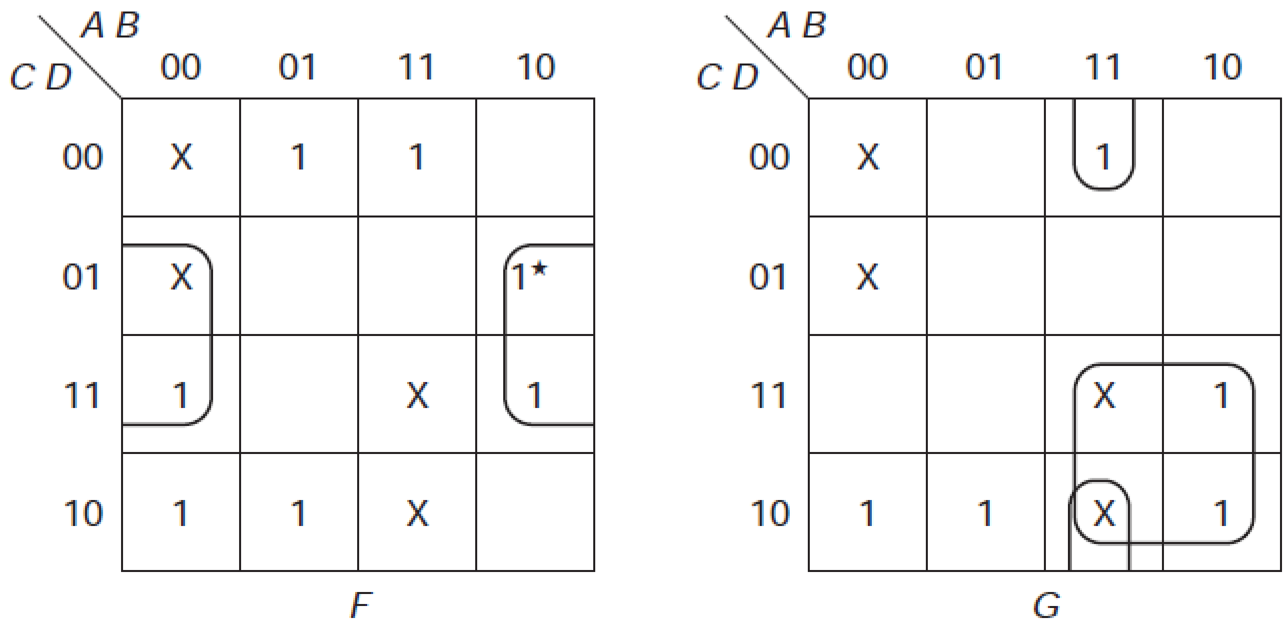
$$G(A, B, C, D) = \Sigma m(2, 6, 10, 11, 12) + \Sigma d(0, 1, 14, 15)$$

- A map of the functions, with the only prime implicant made essential by a 1 that is not shared circled, *B’D*, is shown below



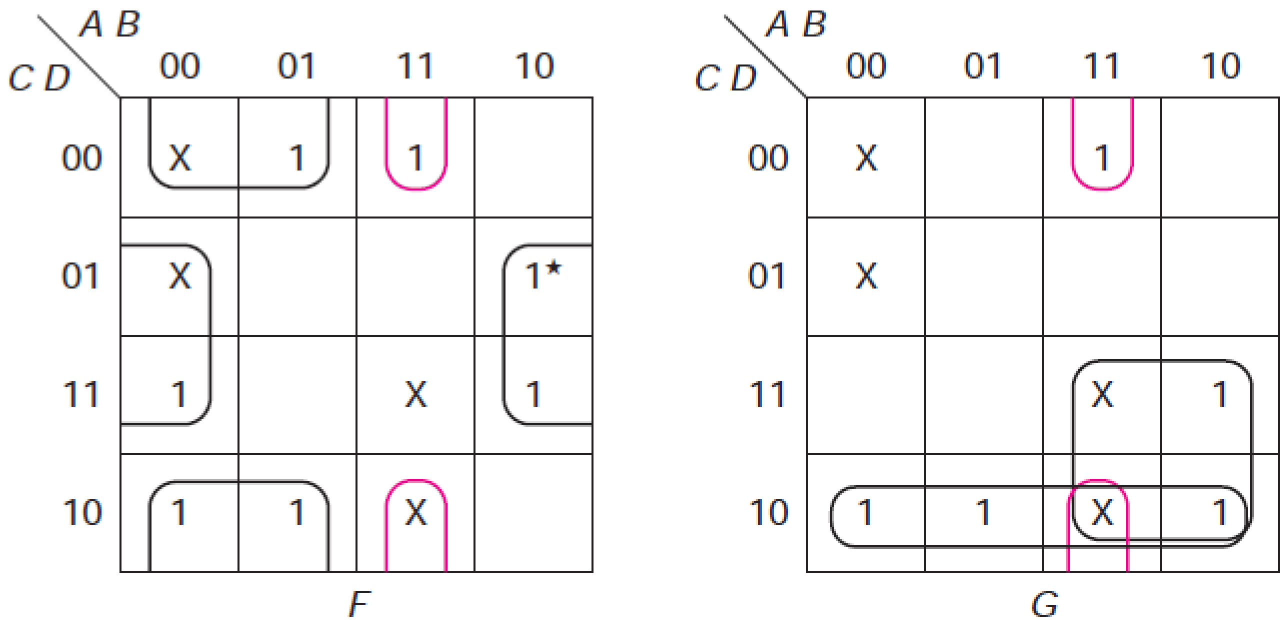
Multiple Output Problems (13/14)

- **Example 3.40 (Cont'd).** Since m_{11} has now been covered in *F*, we must use the essential prime implicant of *G*, *AC*, to cover m_{11}



Multiple Output Problems (14/14)

- **Example 3.40 (Cont'd).** Since we need the term *ABD'* for *G*, one approach is to use it for *F* also



$$F = B'D + ABD' + A'D'$$

$$G = AC + ABD' + CD'$$

- The solution use seven gates and 17 inputs