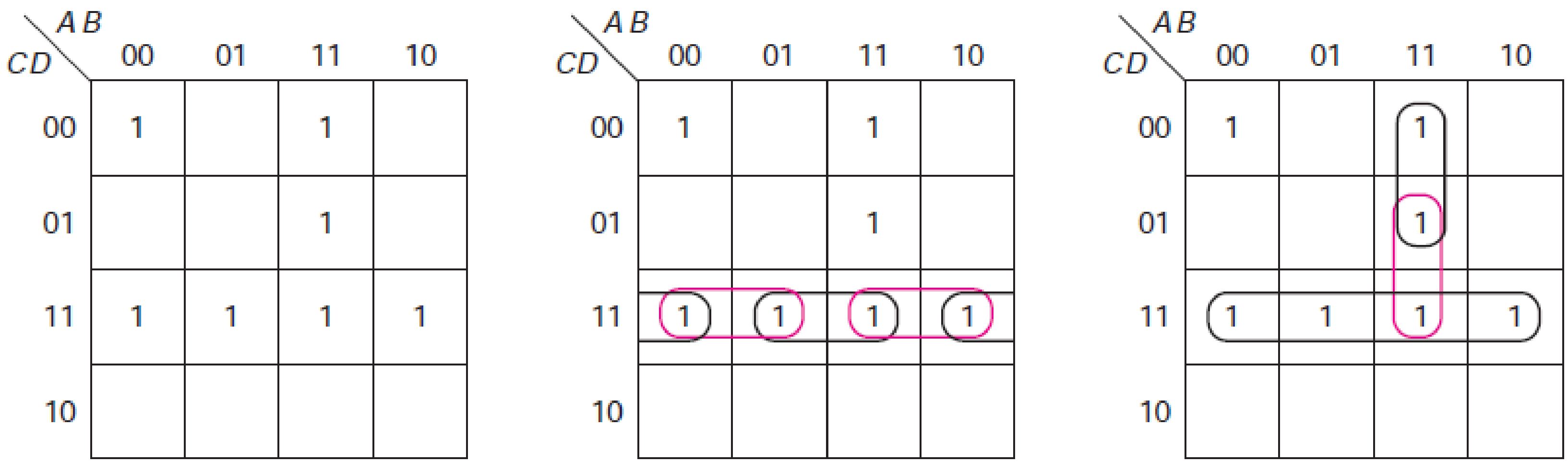
Chapter 3 The Karnaugh Map (II)

Some Terminologies Related to the Karnaugh Map (1/4)

- An *implicant* (涵項) of a function is a product term that can be used in an SOP expression for that function. The function is 1 whenever the implicant is 1
- An implicant is a rectangle of 1, 2, 4, 8,… (any power of 2) 1's. That rectangle may not include any 0's. All minterms are implicants
- We must choose enough implicants such that each of the 1's of the function are included in at least one of these implicants
- We sometimes say that an implicant covers certain minterms

Some Terminologies Related to the Karnaugh Map (2/4)

Map 3.12 A function to illustrate definitions.



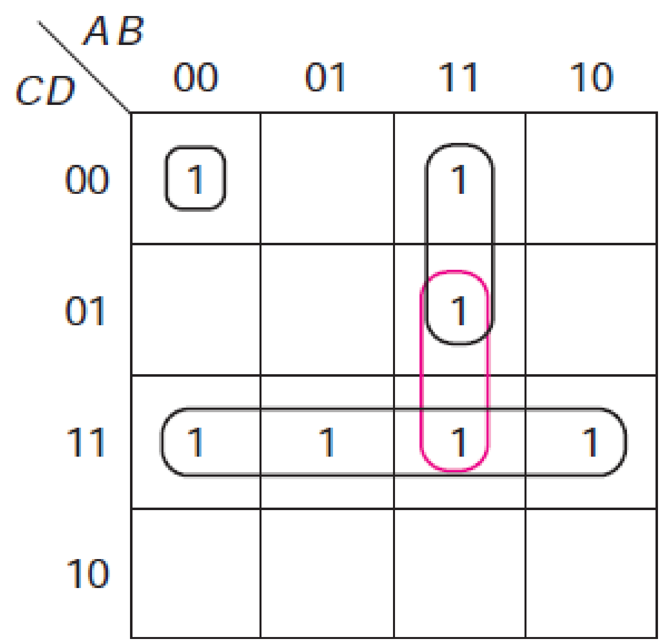
The implicants of F are

Minterms	*Groups of 2*	*Groups of 4*
$A'B'C'D'$	$A'CD$	CD
$A'B'CD$	BCD	
$A'BCD$	ACD	
$ABC'D'$	$B'CD$	
$ABC'D$	ABC'	
$ABCD$	ABD	
$AB'CD$		

Some Terminologies Related to the Karnaugh Map (3/4)

- A *prime implicant* (質涵項) is an implicant that (from the point of view of the map) is not fully contained in any one other implicant
- On Map 3.13, all the prime implicants are circled. They are *A'B'C'D'*, *ABC'*, *ABD*, and *CD*

Map 3.13 Prime implicants.



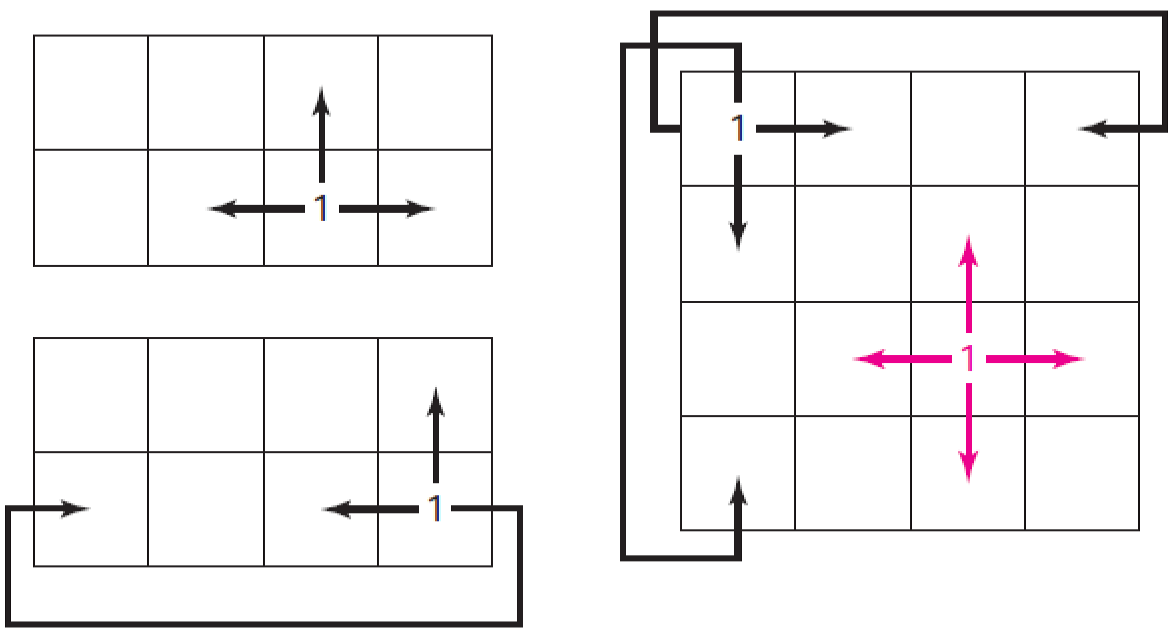
Some Terminologies Related to the Karnaugh Map (4/4)

- An *essential prime implicant* (必要質涵項) is a prime implicant that includes at least one 1 that is not included in any other prime implicant
- The term essential is derived from the idea that we must use that prime implicant in any minimum SOP expression
- In the example of Map 3.13, *A'B'C'D'*, *ABC'*, and *CD* are essential prime implicants; *ABD* is not

Minimum Sum of Product Expressions Using the Karnaugh Map (1/15)

- In the process of finding prime implicants, we will be considering each of the 1's on the map starting with the most isolated 1's
- In an *n*-variable map, each square has *n* adjacent squares

Map 3.14 Adjacencies on three- and four-variable maps.



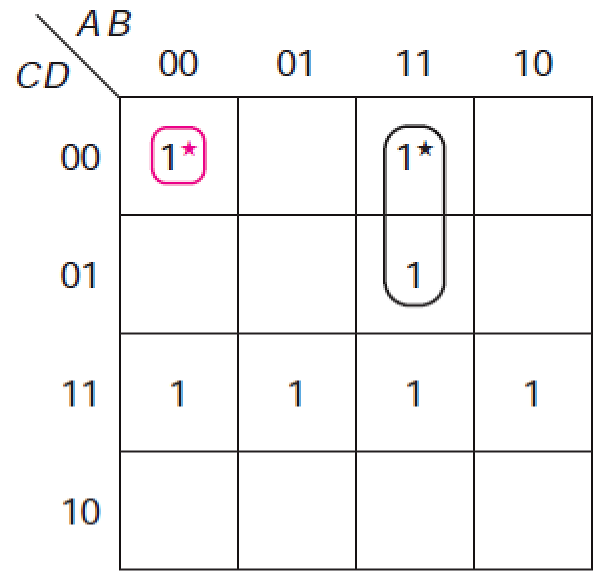
Minimum Sum of Product Expressions Using the Karnaugh Map (2/15)

- Map Method 1

1. Find all essential prime implicants. Circle them on the map and mark the minterms(s) that make them essential with a star (★). Do this by examining each 1 on the map that has not already been circled. It is usually quickest to start with the most isolated 1's
2. Find enough other prime implicants to cover the function. Do this using two criteria:
   a. Choose a prime implicant that covers as many new 1's
   b. Avoid leaving isolated uncovered 1's

Minimum Sum of Product Expressions Using the Karnaugh Map (3/15)

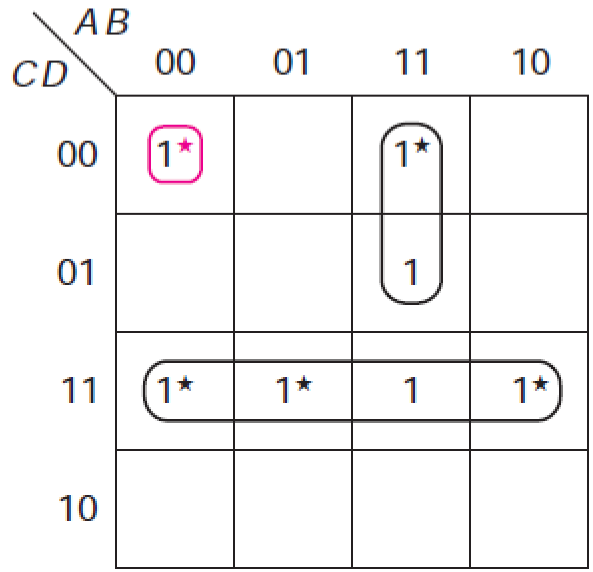
- **Example 3.5** Find the function *F* of the following Karnaugh map.



- As noted, m_0 has no adjacent 1's; therefore, it ($A'B'C'D'$) is a prime implicant. Indeed, it is an essential prime implicant, since no other prime implicant covers this 1

Minimum Sum of Product Expressions Using the Karnaugh Map (4/15)

- **Example 3.5 (Cont'd)** We can cover the function by the groups:



resulting in

$$F = {\color{magenta}A'B'C'D} + ABC' + CD$$

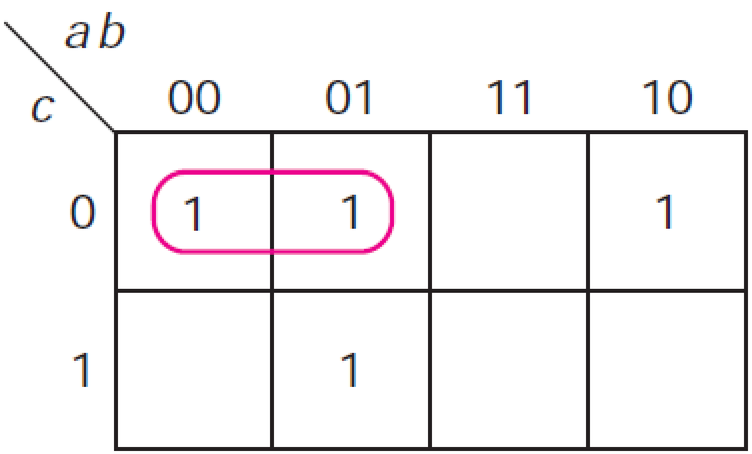
Minimum Sum of Product Expressions Using the Karnaugh Map (5/15)

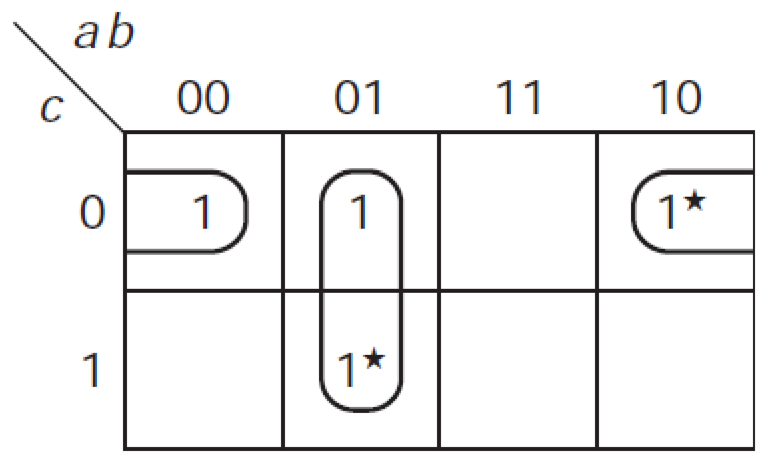
- **Example 3.7** Simplify the function

$$f = a'b'c' + a'bc' + a'bc + ab'c'.$$

We produce the solution

$$f = a'b + b'c'$$

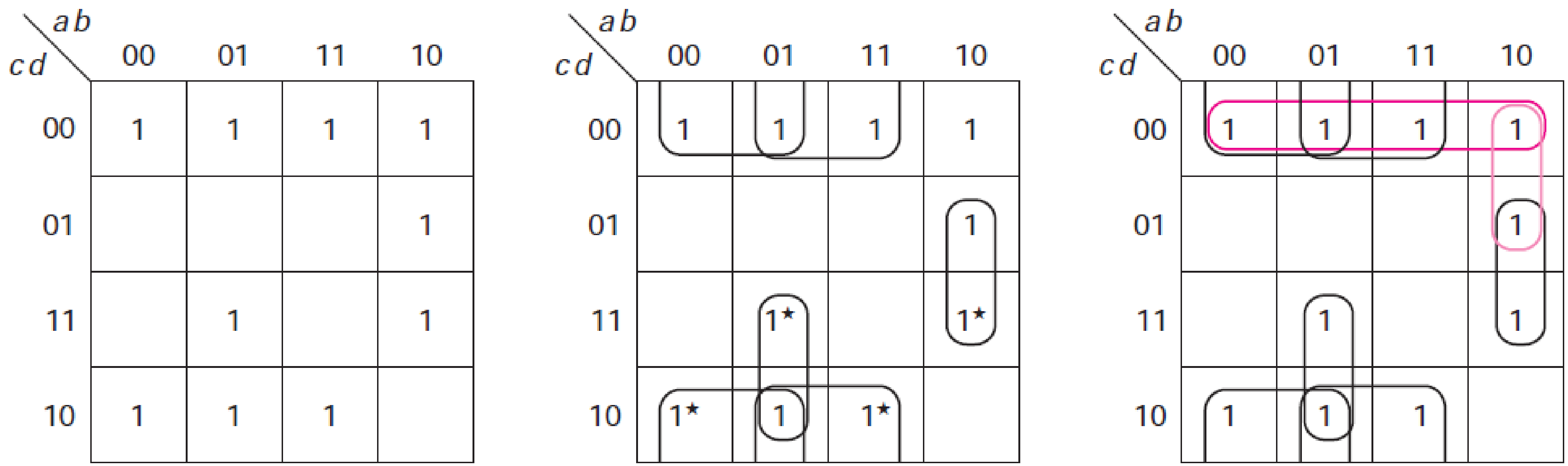




Minimum Sum of Product Expressions Using the Karnaugh Map (6/15)

- **Example 3.8** Simplify the function

$$f(a,b,c,d)=\Sigma m(0,2,4,6,7,8,9,11,12,14).$$



- We obtian

$$f=a'd'+bd'+a'bc+ab'd+c'd'$$

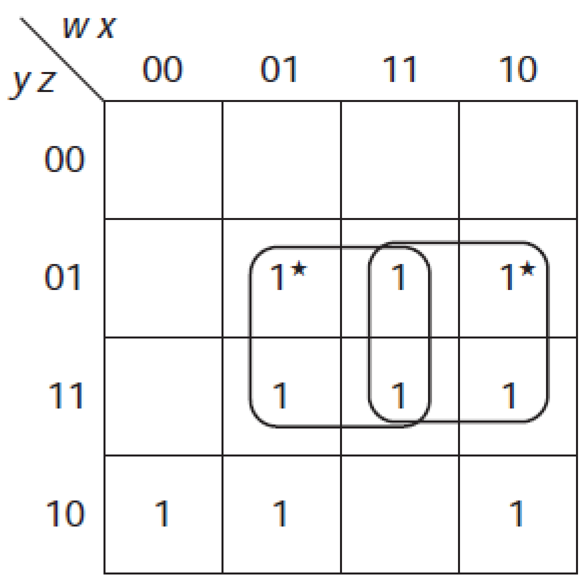
Minimum Sum of Product Expressions Using the Karnaugh Map (7/15)

- **Example 3.11** Simplify the function

$$f(w,x,y,z)=\Sigma m(2,5,6,7,9,10,11,13,15).$$

- The two essential prime implicants are shown on the map, giving

$$g=xz+wz+\ldots$$



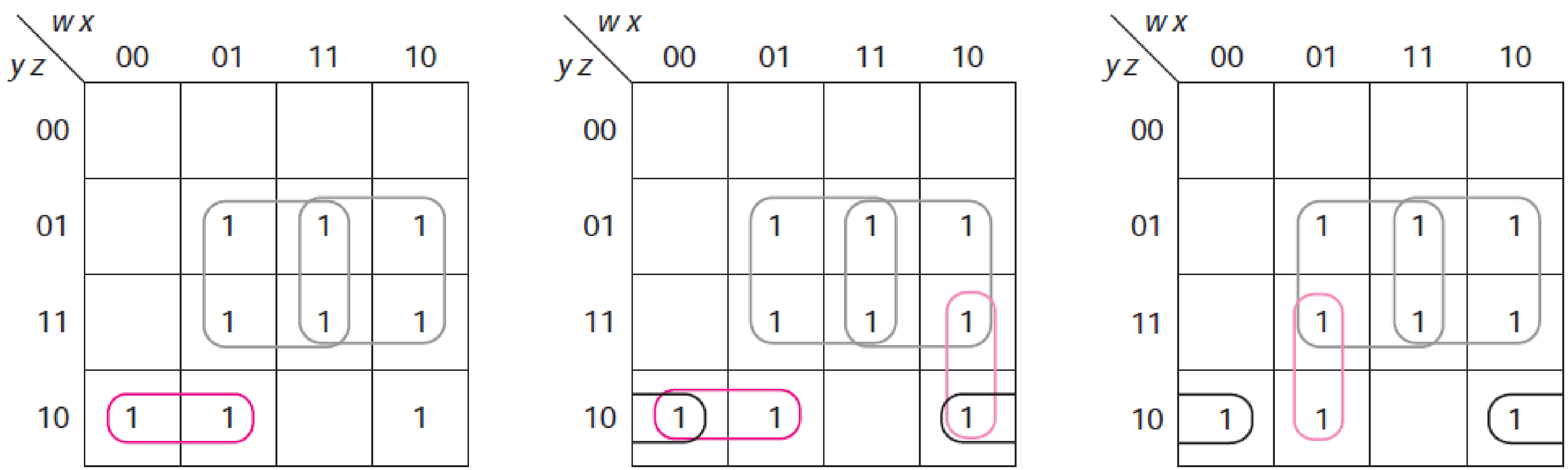
Minimum Sum of Product Expressions Using the Karnaugh Map (8/15)

- **Example 3.11 (Cont'd)** There are three solutions. All three minimum solutions require four terms and 10 literals.

$$g = xz + wz + w'yz' + wx'y$$

$$g = xz + wz + w'yz' + x'yz'$$

$$g = xz + wz + x'yz' + w'xy$$



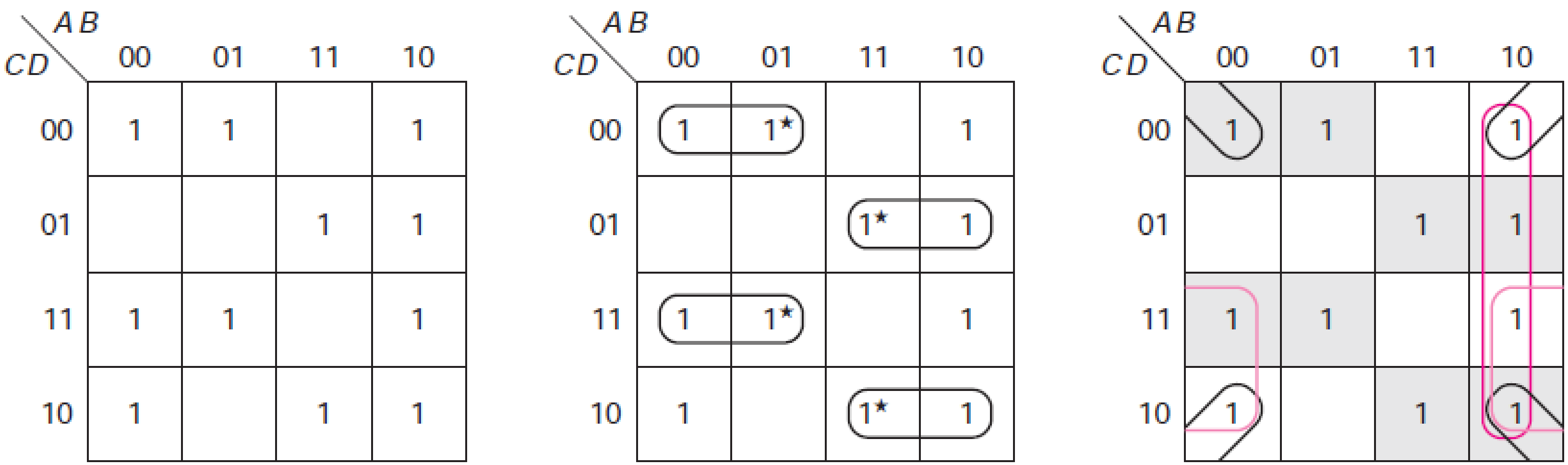
Minimum Sum of Product Expressions Using the Karnaugh Map (9/15)

- **Example 3.11 (Cont'd)** If there are multiple solutions, all minimum solutions must have the same number of terms and literals, if, for example, you find a minimum solution with three terms and seven literals, no solution with four terms is minimum, and no solution with three terms and eight literals is minimum

Minimum Sum of Product Expressions Using the Karnaugh Map (10/15)

- **Example 3.12.** Find the minimum SOP of the function.
- The four essential prime implicants are shown on the second map, levaing three 1's to be covered

$$F = A'C'D' + AC'D + A'CD + ACD' + \cdots$$



Minimum Sum of Product Expressions Using the Karnaugh Map (11/15)

- **Example 3.12. (Cont'd)**
- We have three equally good answers

$$F = A'C'D' + AC'D + A'CD + ACD' + B'D' + AB'$$

$$F = A'C'D' + AC'D + A'CD + ACD' + B'D' + B'C$$

$$F = A'C'D' + AC'D + A'CD + ACD' + AB' + B'C$$

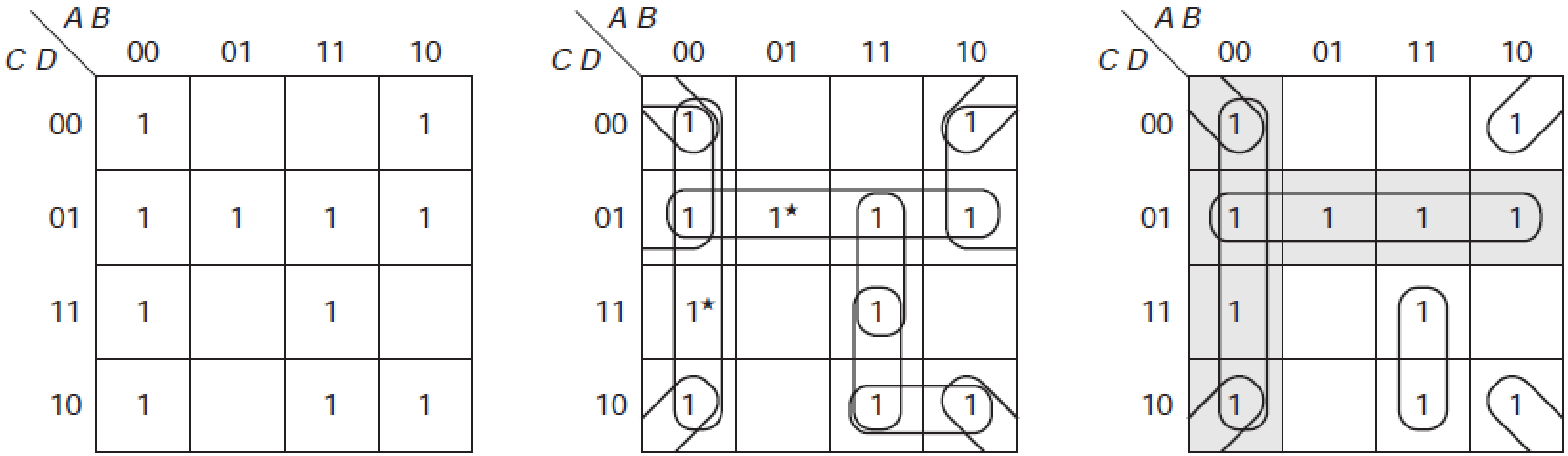
Minimum Sum of Product Expressions Using the Karnaugh Map (12/15)

- Map Method 2

1. Circle all of the prime implicants.
2. Select all essential prime implicants; they are easily identified by finding 1's that have only been circled once.
3. Then choose enough of the other prime implicants (as in Method 1). Of course, these prime implicants have already been identified in step 1.

Minimum Sum of Product Expressions Using the Karnaugh Map (13/15)

- **Example 3.15.** Find the minimum SOP of the function.

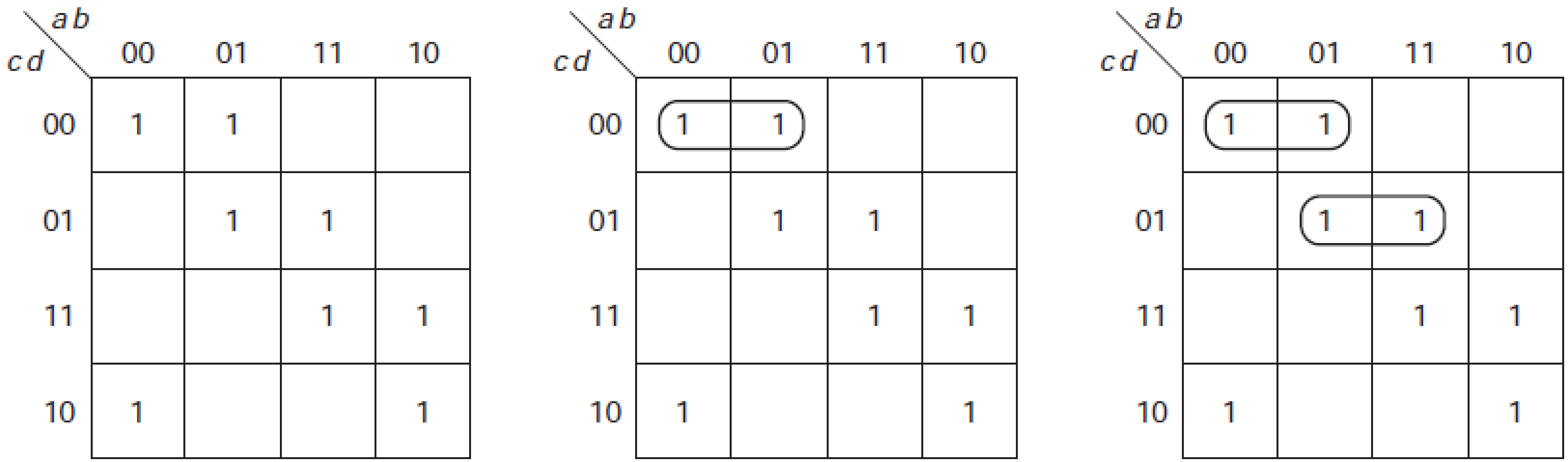


- All the prime implicants have been circled on the center map. By the right map, we need at least two more terms to cover the rest 1's. This produces

$$F = A'B' + C'D + B'D' + ABC$$

Minimum Sum of Product Expressions Using the Karnaugh Map (14/15)

- **Example 3.17.** Find the minimum SOP of the function.



- This example with no essential prime implicants. There are eight 1's; all prime implicants are groups of two. We need at least four terms in a minimum solution

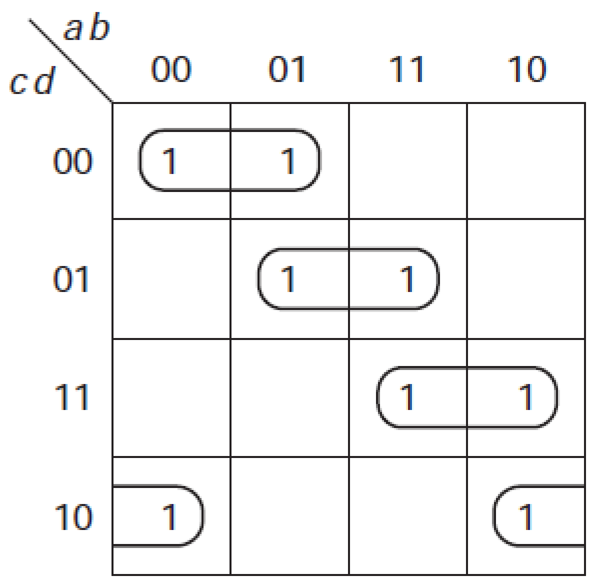
Minimum Sum of Product Expressions Using the Karnaugh Map (15/15)

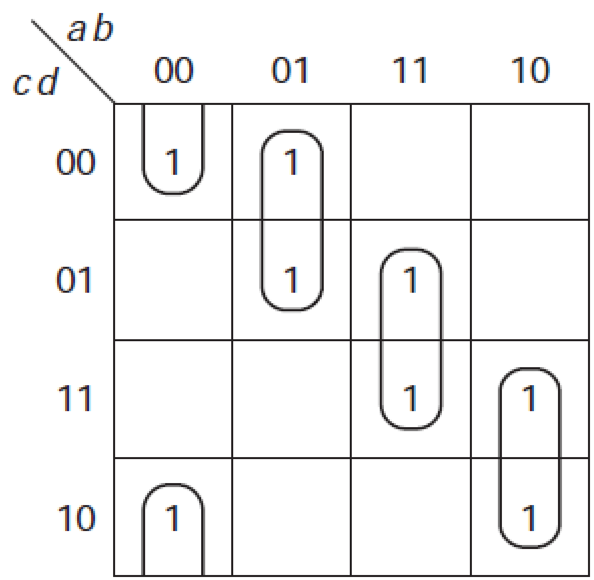
- **Example 3.17 (Cont'd)** There are two solution. They are

$$f=a'c'd'+bc'd+acd+b'cd'$$

and

$$f=a'b'd'+a'bc'+abd+ab'c$$



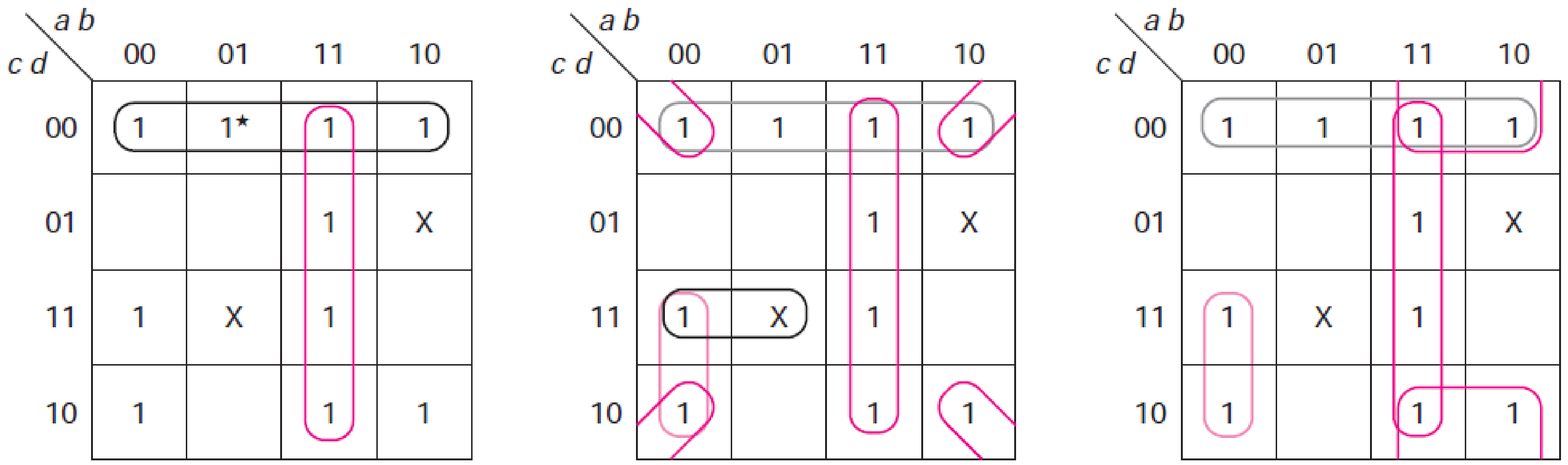


Don’t Cares (1/3)

- Finding minimum solutions for functions with don’t carres does not significantly change the methods we developed. We need to modify slightly the definitions of an implicant and a prime implicant, and an essential prime implicant
- An *implicant* is a rectangle of 1, 2, 4, 8,…1’s or X’s (containing no zeros)
- A *prime implicant* is an implicant not included in any one larger rectangle. Thus, from the point of view of finding prime implicants, X’s (don’t cares) are treated as 1’s
- An *essential prime implicant* is a prime implicant that covers at least one 1, but not covered by any other prime implicant. Don’t cares do not make a prime implicant essential

Don’t Cares (2/3)

- **Example 3.22.** Find the minimum SOP of the function.



- On the first map, we have shown the only essential prime implicant, *c’d’*, and the other group of four that is used in all three solutions, *ab*.

Don't Cares (3/3)

- **Example 3.22 (Cont'd)** On the second map, we have shown two of the solutions, those that utilize *b'd'* as the group of four. On the third map, we have shown the third solution, utilizing *ad'*. We have

$$g_1 = c'd' + ab + b'd' + a'cd$$
$$g_2 = c'd' + ab + b'd' + a'b'c$$
$$g_3 = c'd' + ab + ad' + a'b'c$$

- From the table, it is clear that $g_2 = g_3$, but neither is equal to g_1

	m_7	m_9
g_1	1	0
g_2	0	0
g_3	0	0

Product of Sums (1/5)

- Finding a minimum product of sums expression requires no new theory. The following approach is the simplest:

1. Map the complement of the function. (If there is already a map for the function, replaced all 0's by 1's, all 1's by 0's, and leave X's unchanged)
2. Find the minimum sum of products expression fo the complement of the function
3. Use DeMorgan's theorem to complement that expression, producing a product of sums expression

Product of Sums (2/5)

- **Example 3.25.** Find the minimum POS expression for the function

$$f(a,b,c,d)=\Sigma m(0,1,4,5,10,11,14).$$

- f' must be the sum of all of the other minterms, that is

$$f'(a,b,c,d)=\Sigma m(2,3,6,7,8,9,12,13,15).$$

cd \ ab	00	01	11	10
00	1	1		
01	1	1		
11				1
10			1	1

f

cd \ ab	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

f'

Product of Sums (3/5)

- **Example 3.25. (Cont'd)** There are one minimum solution for *f* and there are two equally good solutions for the sum of products for *f'*

$$f = a'c' + ab'c + acd' \qquad f' = ac' + a'c + abd$$
$$f' = ac' + a'c + bcd$$

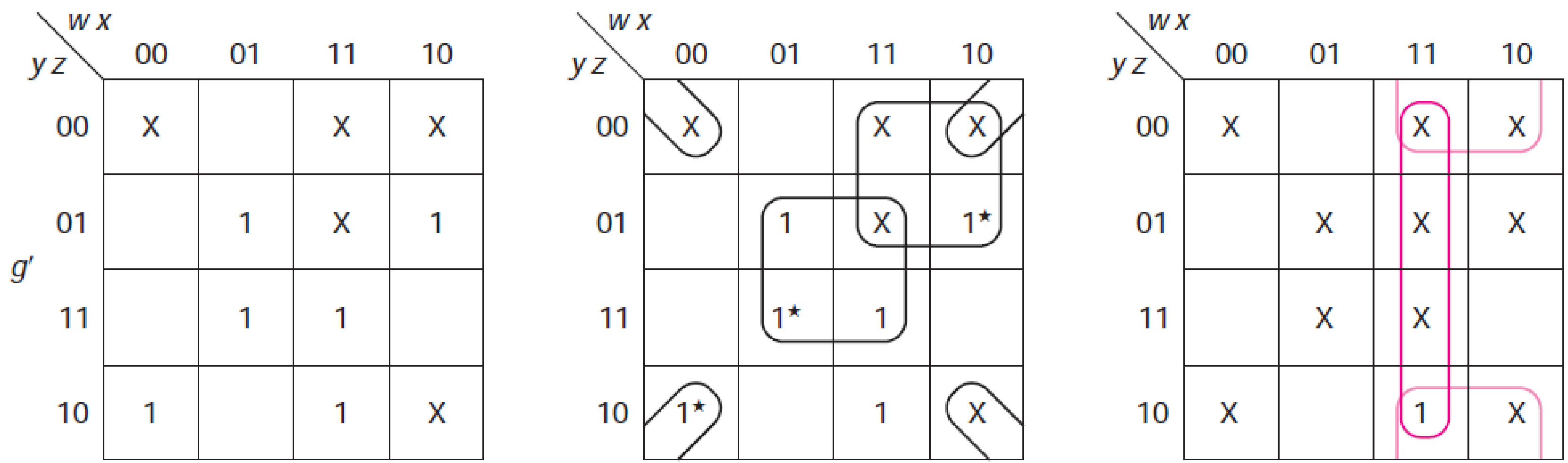
- We can then complement the solutions for *f'* to get the two minimum product of sums solutions for *f*:

$$f = (a' + c)(a + c')(a' + b' + d')$$
$$f = (a' + c)(a + c')(b' + c' + d')$$

Product of Sums (4/5)

- **Example 3.26.** Find all the minimum POS expression for

  g(w,x,y,z)=Σ*m*(1,3,4,6,11)+Σ*d*(0,8,10,12,13).



- The 1’s covered by the essential prime implicants are made don’t cares on the right-hand map

Product of Sums (5/5)

- **Example 3.26 (Cont'd)** We have

$$g' = x'z' + xz + wy' + \begin{Bmatrix} wx \\ wz' \end{Bmatrix}$$

$$g = (x + z)(x' + z')(w' + y)\begin{Bmatrix} (w' + x') \\ (w' + z) \end{Bmatrix}$$