Chapter 7 Analog-to-Digital Conversion (IV)

Waveform Coding (1/1)

- Waveform coding schemes are designed to reproduce the waveform output of the source at the destination with as little distortion as possible
- All attempts are directed at reproducing the source output at the destination with high fidelity. Some basic waveform coding methods include:
  - Pulse code modulation (PCM)
  - Differential pulse code modulation (DPCM)
  - Delta modulation (DM)
  - Adaptive delta modulation (ADM)

Pulse Code Modulation (1/2)

- Pulse code modulation is the simplest and oldest waveform coding scheme. A pulse code modulator consists of three basic sections: a sampler, a quantizer and an encoder
- PCM can be further divided into two types
  - Uniform PCM: the quantizer is a uniform quantizer
  - Nonuniform PCM: the quantizer is nonuniform quantizer that has quantization regions of various sizes

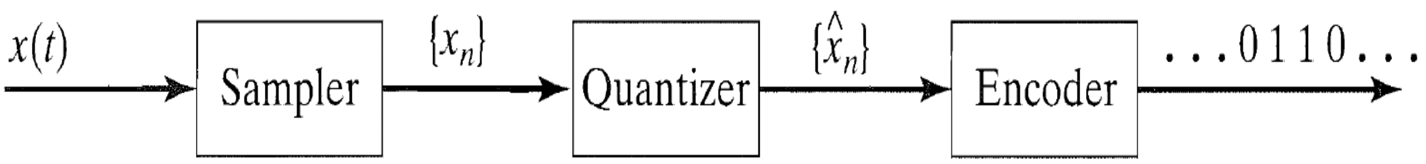

Figure 7.7 Block diagram of a PCM system.

Pulse Code Modulation (2/2)

- In PCM, we make the following assumptions:
  1. The waveform (signal) is bandlimited with a maximum of W. It can be reconstructed at a rate of $f_s=2W$ or higher
  2. The signal is of finite amplitude. In other words, we have, $|x(t)|<x_{max}$
  3. The quantization is done with a large number of quantization levels N, which is a power of 2 ($N=2^\nu$)
- Usually, there exists a filter with bandwidth W prior to the sampler to prevent any components beyond W from entering the sampler. This filter is called the presampling filter. The sampling is done at a rate higher than the Nyquist rate; this allows for some guardband

Uniform PCM (1/6)

- In uniform PCM, we assume that the quantizer is a uniform quantizer. Since the range of the input samples is $[-x_{max}, +x_{max}]$ and the number of quantization levels is N, the length of each quantization region is given by

$$\Delta = \frac{2x_{\max}}{N} = \frac{x_{\max}}{2^{\nu-1}}$$

- The quantized values in uniform PCM are chosen to be the midpoint of the quantization regions; therefore, the error $\tilde{X} = X - Q(X)$ is a random variable taking values uniformly in the interval $(-\Delta/2, +\Delta/2]$. In other words,

$$f_{\tilde{X}}(\tilde{x}) = \begin{cases} \frac{1}{\Delta}, & -\frac{\Delta}{2} \le \tilde{x} \le \frac{\Delta}{2} \\ 0, & otherwise \end{cases}.$$

Uniform PCM (2/6)

- The distortion introduced by quantization (quantization noise) is therefore

$$E[\tilde{X}^2] = \int_{-\frac{\Delta}{2}}^{+\frac{\Delta}{2}} \frac{1}{\Delta}\tilde{x}^2 d\tilde{x} = \frac{\Delta^2}{12} = \frac{x_{\max}{}^2}{3N^2} = \frac{x_{\max}{}^2}{3\times 4^{\nu}}$$

- ν is the number of bits/source sample
- The signal-to-quantization noise ratio becomes

$$SQNR = \frac{P_X}{\overline{E[\tilde{X}^2]}} = \frac{3\times N^2 P_X}{x_{\max}{}^2} = \frac{3\times 4^{\nu} P_X}{x_{\max}{}^2} \qquad (7.4.3)$$

- P_X is the average power in each sample
- SQNR in uniform PCM deteriorates as the dynamic range of the source increases

Uniform PCM (3/6)

- Assume $X(t)$ is a wide-sense stationary process, P_X can be found using any of the following relations:

$$\begin{aligned} P_X &= R_X(\tau)\big|_{\tau=0} \\ &= \int_{-\infty}^{\infty} S_X(f)\,df \\ &= \int_{-\infty}^{\infty} x^2 f_X(x)\,dx \end{aligned}$$

- Since x_{max} is the maximum possible value for X, we always have $P_X = E[X^2] \leqq x_{max}{}^2$. This means $\frac{P_X}{x_{\max}{}^2} \le 1$ (usually $\frac{P_X}{x_{\max}{}^2} << 1$). From (7.4.3) we know that $3N^2 = 3 \times 4^{\nu}$ is an upper bound to the SQNR in uniform PCM

Uniform PCM (4/6)

- From (7.4.3) we can express SQNR in decibels as

$$SQNR\Big|_{dB} \approx 10\log_{10}\frac{P_X}{x_{\max}{}^2} + 6\nu + 4.8$$

We can see that each extra bit (increase in ν by one) increases the SQNR by 6 dB. This is a very useful strategy for estimating how many extra bits are required to achieve a desired SQNR

Uniform PCM (5/6)

- **Example 7.4.1.** What is the resulting SQNR for a signal uniformly distributed on [-1,1], when uniform PCM with 256 levels is employed?
- We have

$$P_X = \int_{-1}^{1} \tfrac{1}{2} x^2 dx = \frac{1}{3}.$$

Therefore, using x_{max}=1 and ν=$\log_2 256$=8, we have

$$SQNR = 3 \times 4^{\nu} \times P_X = 4^{\nu} = 65536 \approx 48.16 \ dB.$$

- If a signal has a bandwidth W, then the minimum number of samples for perfect reconstruction of the signal is given by the sampling theorem, and it is equal to $2W$ samples/sec.

Uniform PCM (6/6)

- If the sample rate is f_s, ν bits are used; therefore, a total of νf_s bits/sec are required for transmission of the PCM signal. In the case of sampling at the Nyquist rate, this is equal to $2\nu W$ bits/sec
- The minimum bandwidth requirement for binary transmission of R bits/sec (or, more precisely, R pulses/sec) is $R/2$ (will be explained in Chap. 9). Therefore, the minimum bandwidth requirement of a PCM system is

$$BW_{req} = \frac{\nu \ f_s}{2},$$

if sampling at the Nyquist rate, gives the minimum bandwidth requirement as

$$BW_{req} = \nu W$$

Nonuniform PCM (1/6)

- As long as the statistics of the input signal are close to the uniform distribution, uniform PCM works fine. However, in coding of certain signals such as speech, the input distribution is far from uniformly distributed
- For a speech waveform, there exists a higher probability for smaller amplitude and a lower probability for larger amplitude
- It makes sense to design a quantizer with more quantization regions at lower amplitudes and fewer quantization regions at larger amplitudes. The resulting quantizer will be a nonuniform quantizer that has quantization regions of various sizes

Nonuniform PCM (2/6)

- The usual method for performing nonuniform quantization is to first pass the samples through a nonlinear element that compresses the large amplitudes (reduces the dynamic range of the signal) and then performs a uniform quantization on the output
- At the receiving end, the inverse (expansion) of this operation is applied to obtain the sampled value. This technique is called *companding* (*compressing-expanding*)

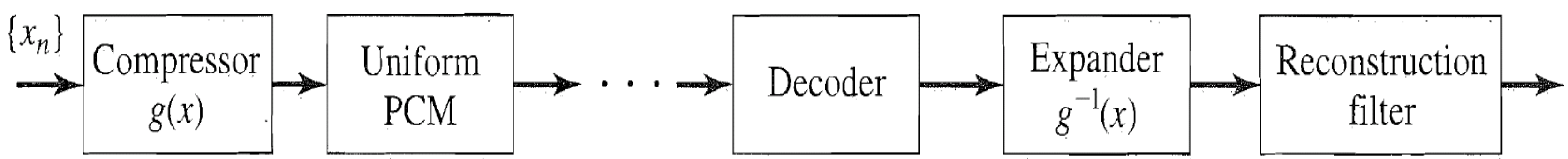

Figure 7.8 Block diagram of a nonuniform PCM system.

Nonuniform PCM (3/6)

- There are two types of companders used for speech coding. One is the μ-law compander, used in the United States and Canada; the other one is the A-law compander
- The μ-law compander employs the logarithmic function at the transmitting side, where $|x| \leqq 1$:

$$g(x) = \frac{\ln(1+\mu|x|)}{\ln(1+\mu)} \operatorname{sgn}(x), \quad \mu \neq 0$$

- If $\mu=0$, then no compression
- The parameter μ controls the amount of compression and expansion. The standard PCM system in the United States and Canada employs a compressor with $\mu=255$ followed by a uniform quantizer with 8 bits/sample

Nonuniform PCM (4/6)

- Use of a compander in this system improves the performance of the system by about 24 dB

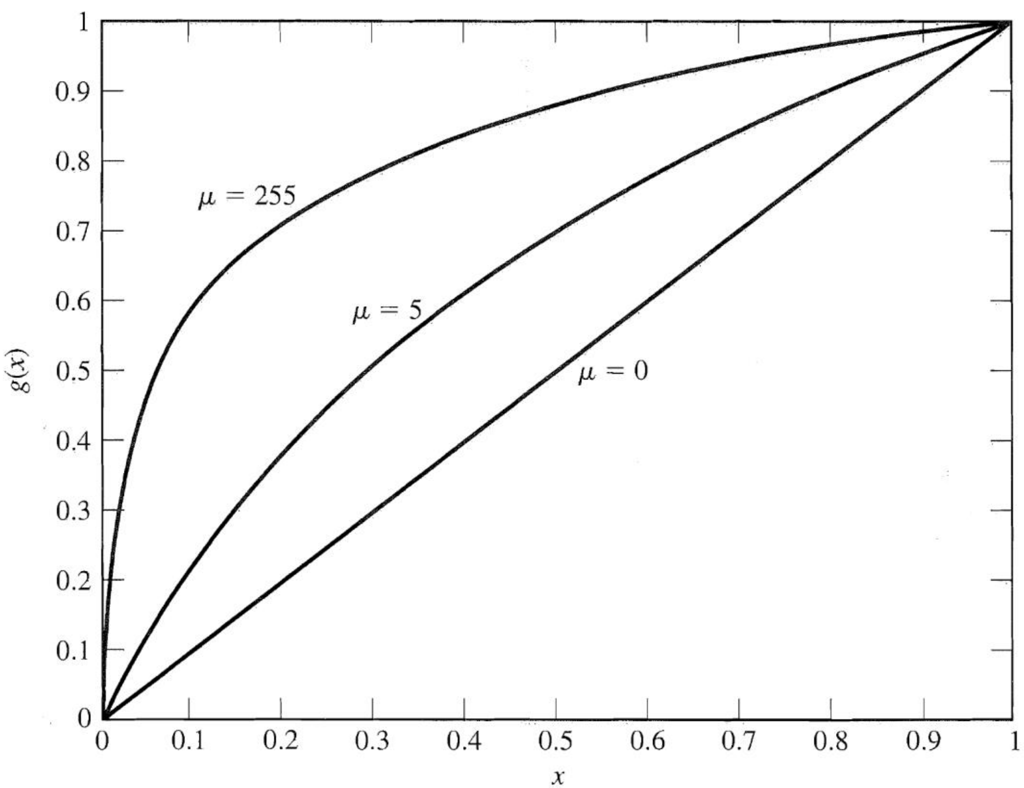

Figure 7.9 A graph of μ-law compander characteristics.

Nonuniform PCM (5/6)

- The second widely used logarithmic compressor is the A-law compander, mainly used in Europe. The characteristics of this compander are given by

$$g(x)=\begin{cases}\dfrac{A|x|}{1+\ln(A)}, & ,0\le|x|<\frac{1}{A}\\[2ex] \dfrac{\operatorname{sgn}(x)(1+\ln(A|x|))}{1+\ln(A)}, & \frac{1}{A}\le|x|\le 1\end{cases},$$

where A is chosen to be 87.56. The performance of this compander is comparable to the performance of the μ-law compander

Nonuniform PCM (6/6)

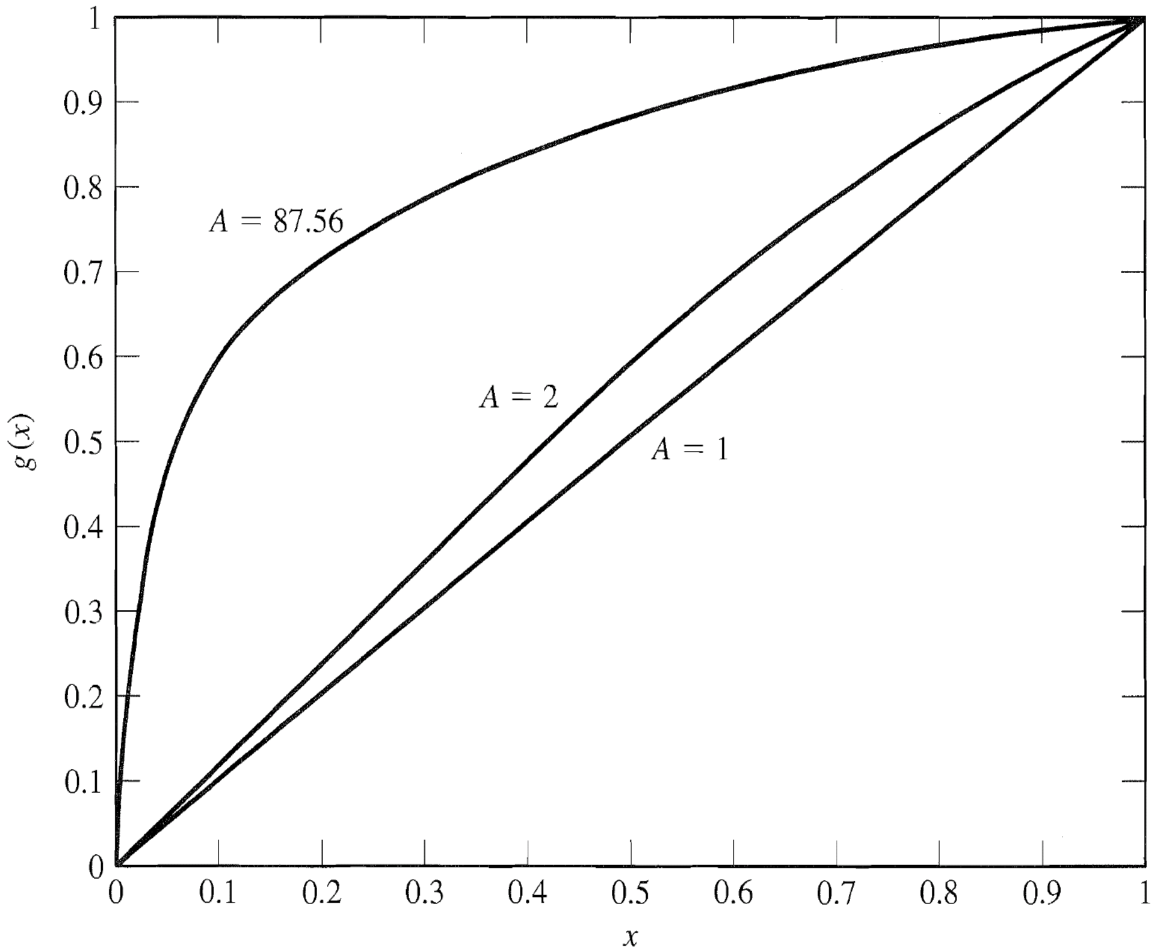

Figure 7.10 A graph of A-law compander characteristics.

Differential Pulse Code Modulation (DPCM) (1/4)

- PCM samples are regarded as independent with each other. In fact, they are correlated as long as the spectrum of the sampled process is flat within its bandwidth. This means that the previous samples give some information about the next sample; thus, this information can be employed to improve the performance of the PCM system
- For instance, if the previous sample values were small, and there is a high probability that the next sample value will be small as well, then it is not necessary to quantize a wide range of values to achieve a good performance

Differential Pulse Code Modulation (DPCM) (2/4)

- In DPCM, the difference between two adjacent samples is quantized. Because two adjacent samples are highly correlated, their difference has small variations
- To achieve a certain level of performance, fewer levels (and therefore fewer bits) are required to quantize it. This means DPCM can achieve performance levels at lower bit rates than PCM

Differential Pulse Code Modulation (DPCM) (3/4)

- $\boxed{T=1}$: a memory to store data for one sample period delay
- Initially, the encoder memory and the decoder memory are set to zeros. For instance, $\hat{Y}'_{-1} = \hat{X}_{-1} = 0$, then for all n we have $\hat{Y}'_n = \hat{X}_n$
- Note that $\hat{Y}'_{n-1} \cong X_{n-1}$
- For n=0, we have $Y_0 = X_0 - \hat{Y}'_{-1} = X_0$, $\hat{Y}_0 = Q(Y_0)$,
  $\hat{X}_0 = \hat{Y}_0 + \hat{X}_{-1} = \hat{Y}_0 = Q(Y_0)$
- For n=1, we have $\hat{X}_1 = \hat{Y}_1 + \hat{X}_0 = \hat{Y}_1 + Q(Y_0) = Q(Y_1) + Q(Y_0)$

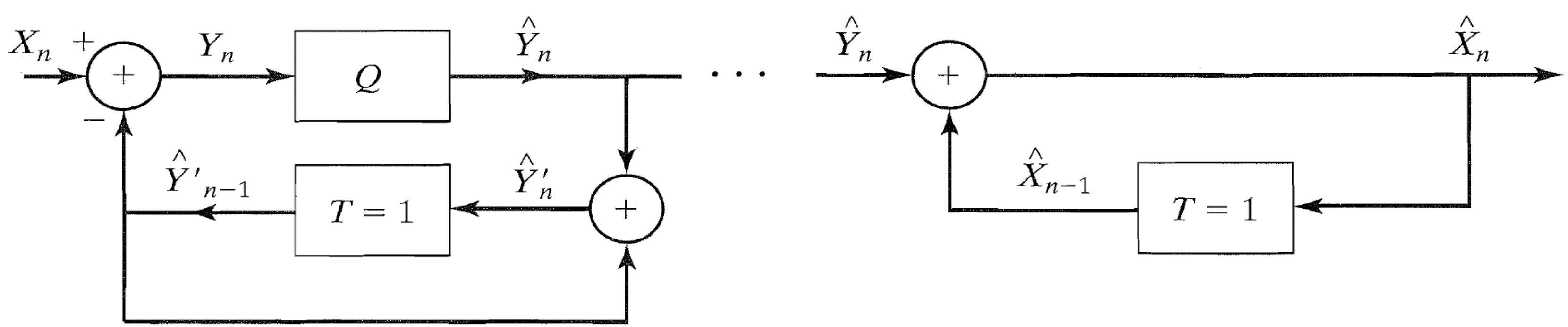

Figure 7.11 A simple DPCM encoder and decoder.

Differential Pulse Code Modulation (DPCM) (4/4)

- The advantage of this design is the accumulation of quantization noise is prevented
- The range of variation of Y_n is usually much smaller than that of X_n; therefore, Y_n can be quantized with fewer bits

Delta Modulation (DM) (1/6)

- Delta modulation is a simplified version of the DPCM system. In delta modulation, the quantizer is a one-bit (two-level) quantizer with magnitudes $\pm\Delta$

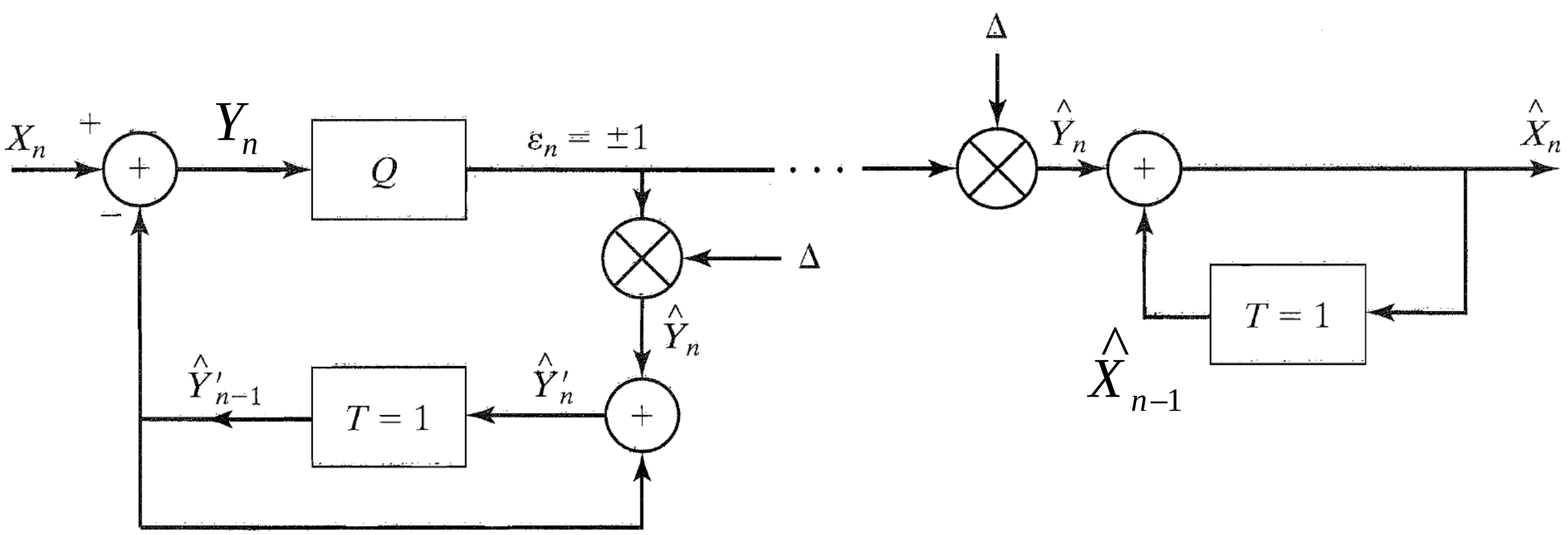

Figure 7.12 Delta modulation.

Delta Modulation (DM) (2/6)

- In delta modulation only one bit per sample is employed, so the quantization noise will be high unless the dynamic range of Y_n is very low. This means that X_n and X_{n-1} must have a very high correlation coefficient
- To have a high correlation between X_n and X_{n-1}, we have to sample at rates much higher than the Nyquist rate. Therefore, in DM, the sampling rate is usually much higher than the Nyquist rate, but since the number of bits per sample is only one, the total number of bits per second required to transmit a waveform is lower than that of a PCM system

Delta Modulation (DM) (3/6)

- A major advantage of delta modulation is the very simple structure of the system. At the receiving end, we have the following relation for the reconstruction of $\hat{X}_n$:

$$\hat{X}_n - \hat{X}_{n-1} = \hat{Y}_n$$

- Solving this equation for $\hat{X}_n$, and assuming zero initial conditions, we obtain

$$\begin{aligned}
\hat{X}_n - \hat{X}_{n-1} &= \hat{Y}_n \\
\hat{X}_{n-1} - \hat{X}_{n-2} &= \hat{Y}_{n-1} \\
&\vdots \\
\hat{X}_0 - \hat{X}_{-1} &= \hat{Y}_0 \\
\Rightarrow \hat{X}_n - \hat{X}_{-1} &= \sum_{i=0}^{n} \hat{Y}_i, \quad \hat{X}_{-1} = 0 \\
\Rightarrow \hat{X}_n &= \sum_{i=0}^{n} \hat{Y}_i \qquad (7.4.15)
\end{aligned}$$

Delta Modulation (DM) (4/6)

- This means that to obtain $\hat{X}_n$, we only have to accumulate the values of $\hat{Y}_n$. If the sampled values are represented by impulses, the accumulator will be a simple integrator. This simplifies the block diagram of a DM system, as shown in Fig. 7.13

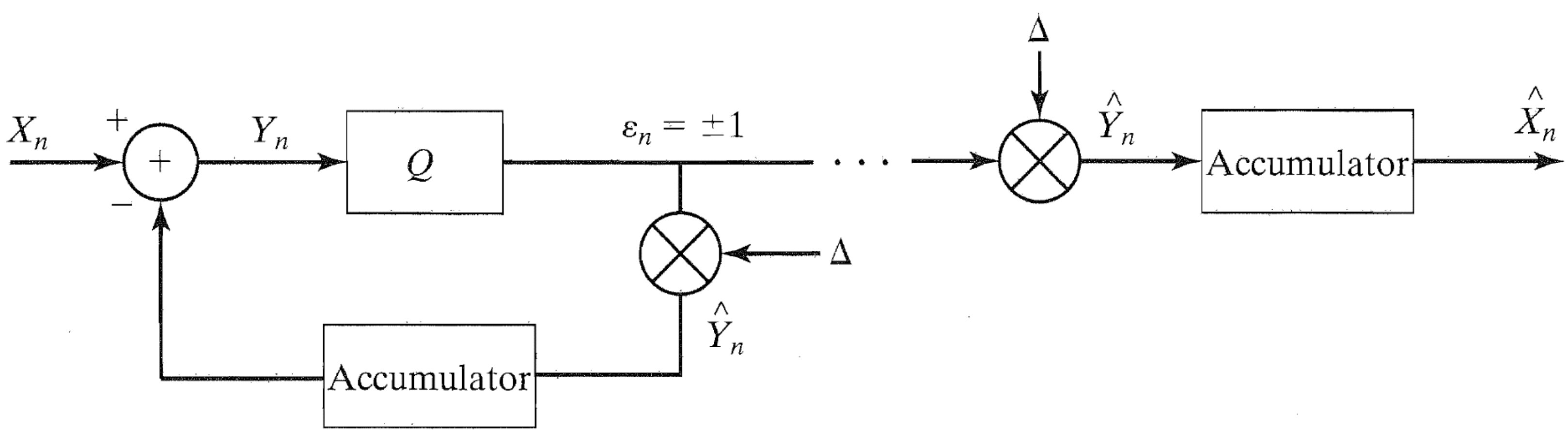

Figure 7.13 Delta modulation with integrators.

Delta Modulation (DM) (5/6)

- Large values of Δ cause the modulator to follow rapid changes in the input signal; but cause excessive quantization noise when the input changes slowly. This case is shown in Fig. 7.14. For large Δ, when the input varies slowly, a large quantization noise occurs; this is known as *granular noise*

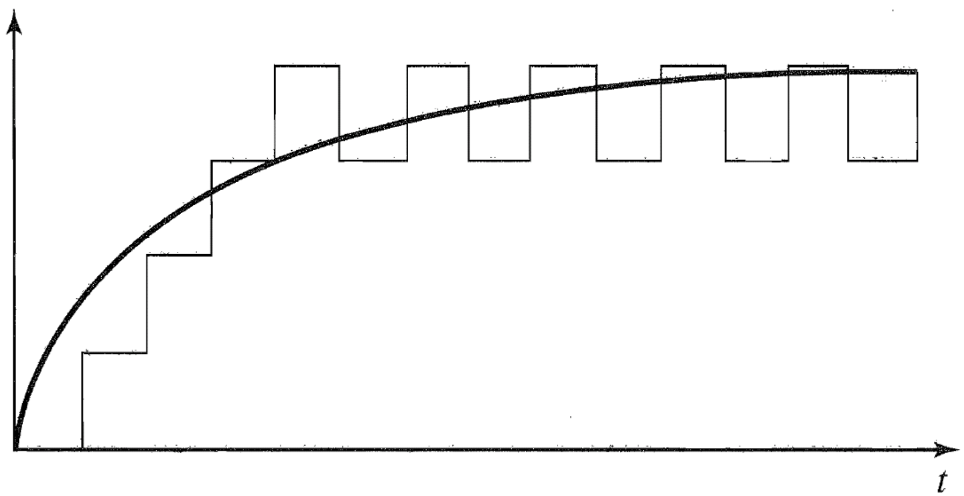

Figure 7.14 Large Δ and granular noise.

Delta Modulation (DM) (6/6)

- The case of a too small Δ is shown in Fig. 7.15. In this case, we have a problem with rapid changes in input. This type of distortion is called *slope overload distortion*

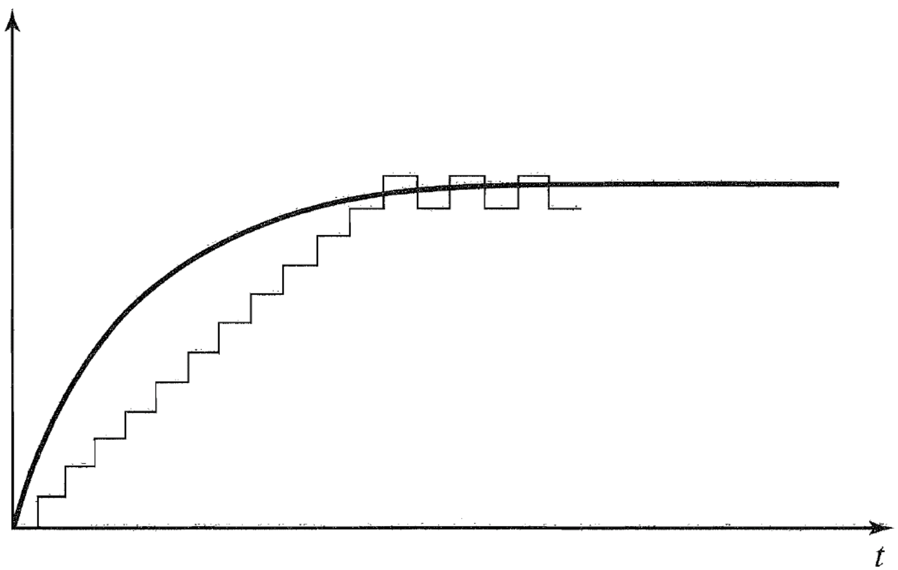

Figure 7.15 Small Δ and slope overload distortion.

Adaptive Delta Modulation (ADM) (1/2)

- If the input tends to change rapidly, the step size must be large so that the output can follow the input quickly and no slope overload distortion results
- When the input is slowly varying, the step size changed to a small value to prevent granular noise as shown in Fig. 7.16

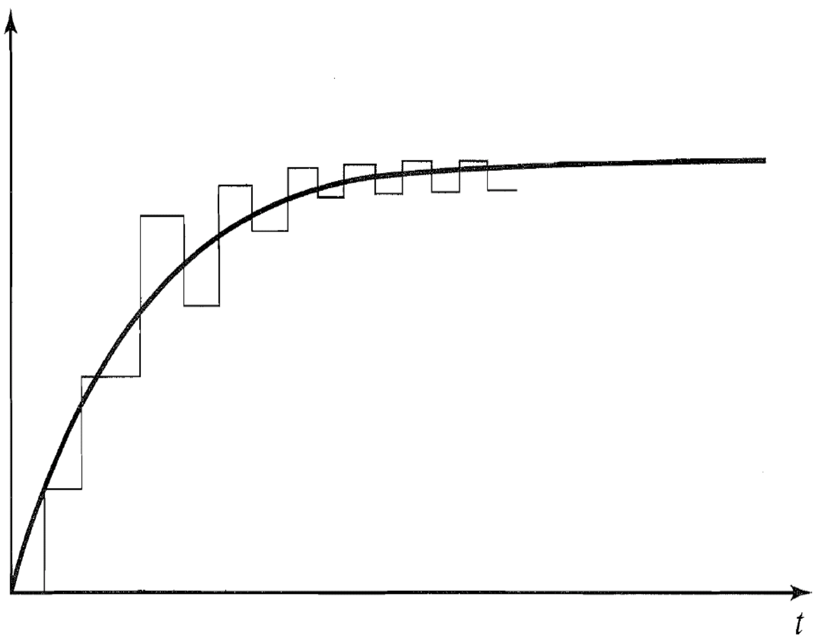

Figure 7.16 Performance of adaptive delta modulation.

Adaptive Delta Modulation (ADM) (2/2)

- If the two successive outputs have the same sign, the step size should be increased; if they are of opposite signs, it should be decreased
- A particularly simple rule to change the step size is given by

$$\Delta_n = \Delta_{n-1} K^{\varepsilon_n \times \varepsilon_{n-1}},$$

where ε_n is the output of the quantizer before being scaled by the step size and K is some constant larger than one

- It has been verified that in the 20-60 kbps range, with a choice of K=1.5, the performance of adaptive delta modulation systems is 5-10 dB better than the performance of delta modulation when applied to speech sources