

Chapter 3 The Karnaugh Map (I)

Introduction to the Karnaugh Map

(1/13)

- In this chapter, we will examine an approach that is easier to implement, the Karnaugh map (sometimes referred to as a K-map). This is a graphical approach to finding suitable product terms for use in sum of product expressions
- This tool was introduced in 1953 by Maurice Karnaugh
- Although there is no guarantee of finding a minimum solution, the methods nearly always produce a minimum
- The Karnaugh map consists of one square for each possible minterm in a function. Thus, a two-variable map has 4 squares, a three-variable map has 8 squares, and a four-variable map has 16 squares

Introduction to the Karnaugh Map (2/13)

- When we plot a function, we put a 1 in each square corresponding to a minterm that is included in the function, and put a 0 in or leave blank squares not included in the function
- Put an X in the square for which the minterm is a don't care

Map 3.2 Plotting functions.

<i>a</i>		
<i>b</i>	0	1
0	1	
1		1

$$f(a, b) = \sum m(0, 3)$$

	<i>A</i>	
	0	1
<i>B</i>		
0	1	X
1		1

$$g(A, B) = \sum m(0, 3) + \sum d(2)$$

Introduction to the Karnaugh Map (3/13)

- Three-variable maps have eight squares
- Both of the following two maps are workable

		<i>AB</i>			
		00	01	11	10
<i>C</i>	0	0	2	6	4
	1	1	3	7	5

		<i>BC</i>			
		00	01	11	10
<i>A</i>	0	0	1	3	2
	1	4	5	7	6

Introduction to the Karnaugh Map (4/13)

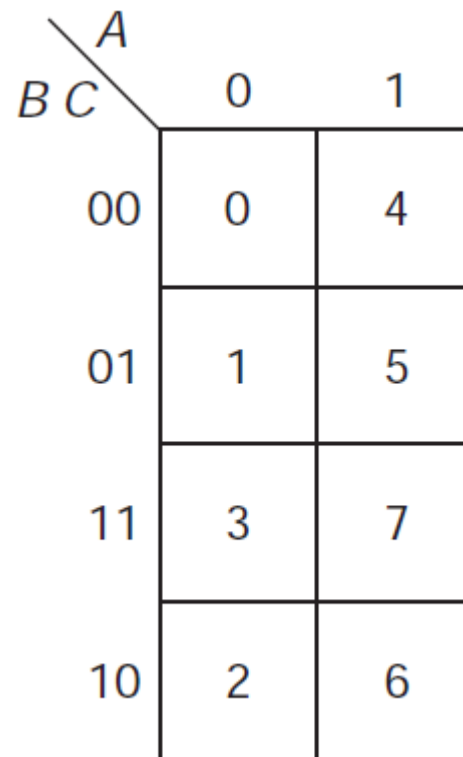
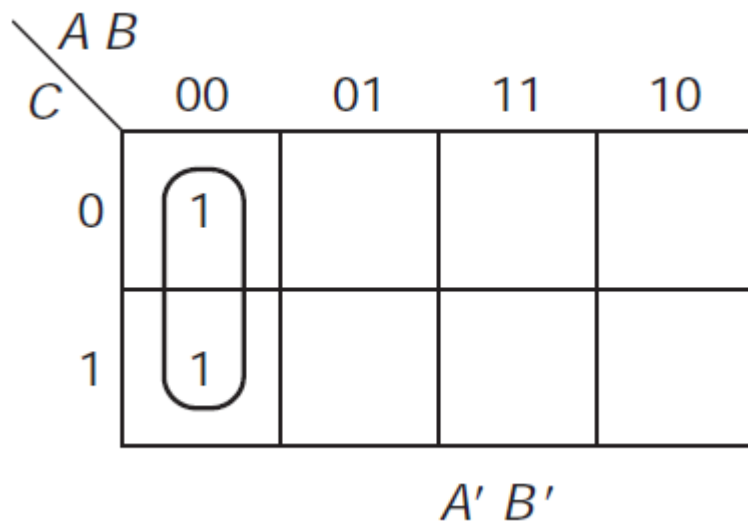
- Map 3.4 is an incorrect K-map because it cannot further combine adjacent minterms into a more compact algebraic format. For example, $m_2 + m_4$ cannot be further simplified.

Map 3.4 Incorrect arrangement of the map.

		<i>A B</i>			
		00	01	10	11
<i>C</i>	0	0 <i>A' B C'</i>	2 <i>A' B C'</i>	4 <i>A B' C'</i>	6
	1	1	3	5	7

Introduction to the Karnaugh Map (5/13)

- The K-map can be drawn in a vertical orientation. Thus, both the following two maps are feasible and will produce the same results. The key is to let adjacent squares differ by only one bit



Introduction to the Karnaugh Map (6/13)

- In reading the map, it is useful to label the pairs of columns (in those arrangements where there are four columns) as shown in Map 3.7

Map 3.7 Map with columns labeled.

$A \ B$	00	01	11	10
0	0	2	6	4
1	1	3	7	5

The diagram shows a 2x4 grid representing a Karnaugh map. The columns are labeled with binary pairs: 00, 01, 11, and 10. A bracket above the columns 01 and 11 is labeled 'B'. The rows are labeled with binary values: 0 and 1. A bracket below the columns 01 and 11 is labeled 'A'. The cells contain the numbers 0, 2, 6, 4 in the top row and 1, 3, 7, 5 in the bottom row.

Introduction to the Karnaugh Map

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Map 3.8 The four-variable map.

$CD \backslash AB$	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

$CD \backslash AB$	00	01	11	10
00	$A'B'C'D'$	$A'BC'D'$	$ABC'D'$	$AB'C'D'$
01	$A'B'CD$	$A'BCD$	$ABC'D$	$AB'C'D$
11	$A'B'CD$	$A'BCD$	$ABCD$	$AB'CD$
10	$A'B'CD'$	$A'BCD'$	$ABCD'$	$AB'CD'$

Introduction to the Karnaugh Map (8/13)

- **Example 3.2** Simplify $m_{13} + m_9$, $m_3 + m_{11}$, $m_0 + m_2$ using a K-map.

$$m_{13} + m_9: \quad ABC'D + AB'C'D = AC'D$$

$$m_3 + m_{11}: \quad A'B'CD + AB'CD = B'CD$$

$$m_0 + m_2: \quad A'B'C'D' + A'B'CD' = A'B'D'$$

	<i>AB</i>			
<i>CD</i>	00	01	11	10
00				
01			1	1
11				
10				

AC'D

	<i>AB</i>			
<i>CD</i>	00	01	11	10
00				
01				
11	1			1
10				

B'CD

	<i>AB</i>			
<i>CD</i>	00	01	11	10
00	1			
01				
11				
10	1			

A'B'D'

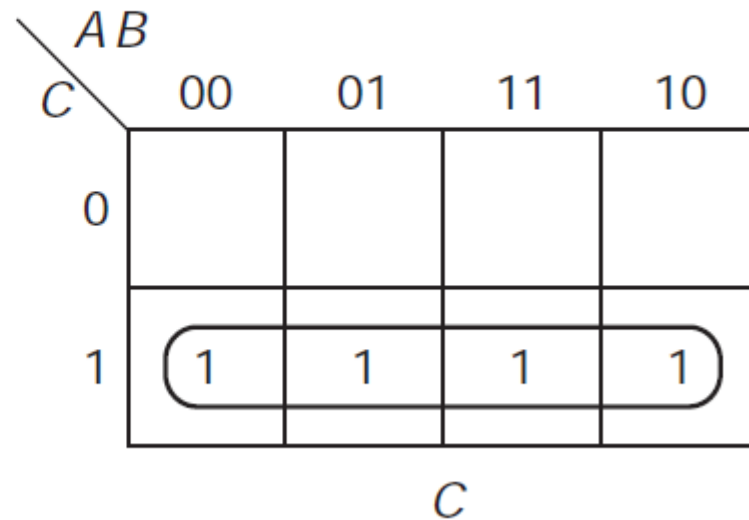
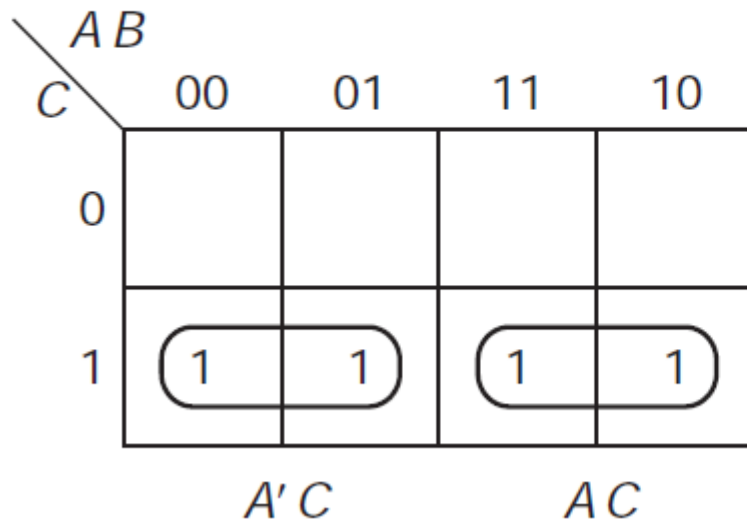
Introduction to the Karnaugh Map (9/13)

- Consider the algebra

$$A'C + AC = C.$$

We can simplify it using a 3-variable K-map

Map 3.9 A group of four 1's.



Introduction to the Karnaugh Map (10/13)

- We can extend P9 to obtain

$$\mathbf{P9aa.} \quad a'b' + a'b + ab + ab' = 1$$

$$\mathbf{P9bb.} \quad (a' + b')(a' + b)(a + b)(a + b') = 0$$

- **Proof of P9aa**

By expanding the equation

$$(a' + a)(b' + b) = 1,$$

we have

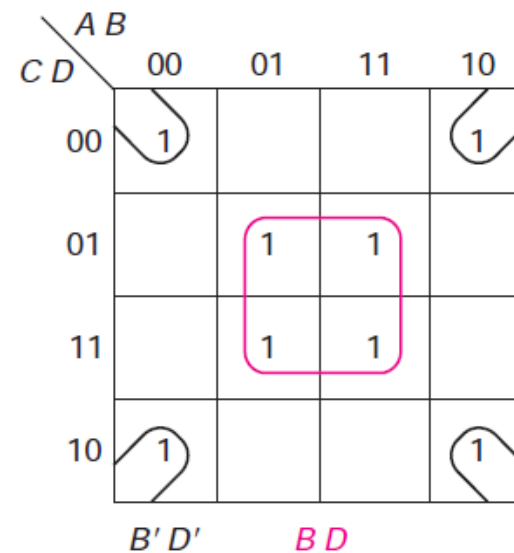
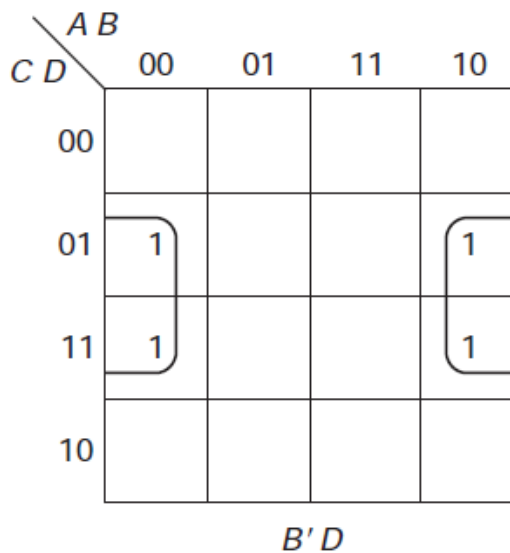
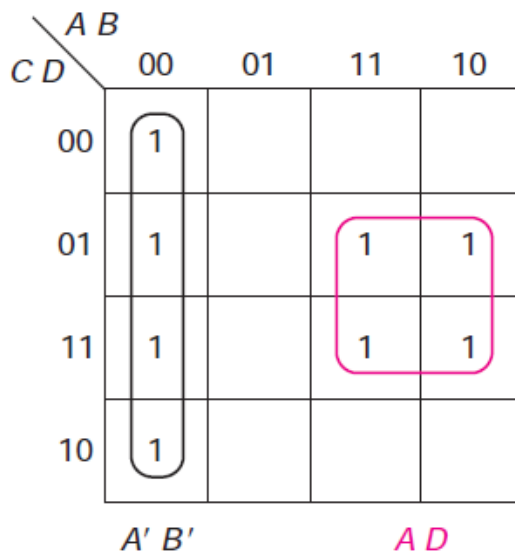
$$a'b' + a'b + ab + ab' = 1.$$

- **Proof of P9bb**

Complement both sides of P9aa, we get P9bb.

Introduction to the Karnaugh Map (11/13)

- Some K-map examples of groups of four minterms



Introduction to the Karnaugh Map (12/13)

- Some examples of groups of eight minterms

<i>CD</i> \ <i>AB</i>	00	01	11	10
00	1	1		
01	1	1		
11	1	1		
10	1	1		

A 4x4 Karnaugh map with variables A and B on the horizontal axis and C and D on the vertical axis. The columns are labeled 00, 01, 11, and 10. The rows are labeled 00, 01, 11, and 10. A group of eight minterms is circled, consisting of all cells where C=0 and D=0, and all cells where C=1 and D=1.

<i>CD</i> \ <i>AB</i>	00	01	11	10
00	1	1	1	1
01				
11				
10	1	1	1	1

A 4x4 Karnaugh map with variables A and B on the horizontal axis and C and D on the vertical axis. The columns are labeled 00, 01, 11, and 10. The rows are labeled 00, 01, 11, and 10. Two groups of four minterms are circled: one group in the top row (C=0, D=0) and one group in the bottom row (C=1, D=1).

Introduction to the Karnaugh Map (13/13)

- **Example 3.3.** Map the function using a K-map, where

$$F = AB' + AC + A'BC'$$

