Chapter 3 The Karnaugh Map (I)

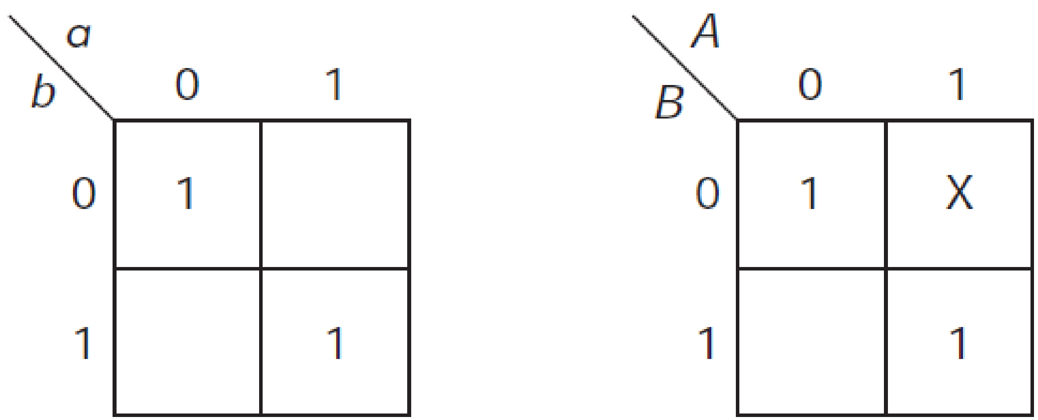
Introduction to the Karnaugh Map (1/13)

- In this chapter, we will examine an approach that is easier to implement, the Karnaugh map (sometimes referred to as a K-map). This is a graphical approach to finding suitable product terms for use in sum of product expressions
- This tool was introduced in 1953 by Maurice Karnaugh
- Although there is no guarantee of finding a minimum solution, the methods nearly always produce a minimum
- The Karnaugh map consists of one square for each possible minterm in a function. Thus, a two-variable map has 4 squares, a three-variable map has 8 squares, and a four-variable map has 16 squares

Introduction to the Karnaugh Map (2/13)

- When we plot a function, we put a 1 in each square corresponding to a minterm that is included in the function, and put a 0 in or leave blank squares not included in the function
- Put an X in the square for which the minterm is a don't care

Map 3.2 Plotting functions.



$f(a, b) = \Sigma m\,(0, 3)$ $\qquad$ $g(A, B) = \Sigma m\,(0, 3) + \Sigma d\,(2)$

Introduction to the Karnaugh Map (3/13)

- Three-variable maps have eight squares
- Both of the following two maps are workable

C \ AB	00	01	11	10
0	0	2	6	4
1	1	3	7	5

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

Introduction to the Karnaugh Map (4/13)

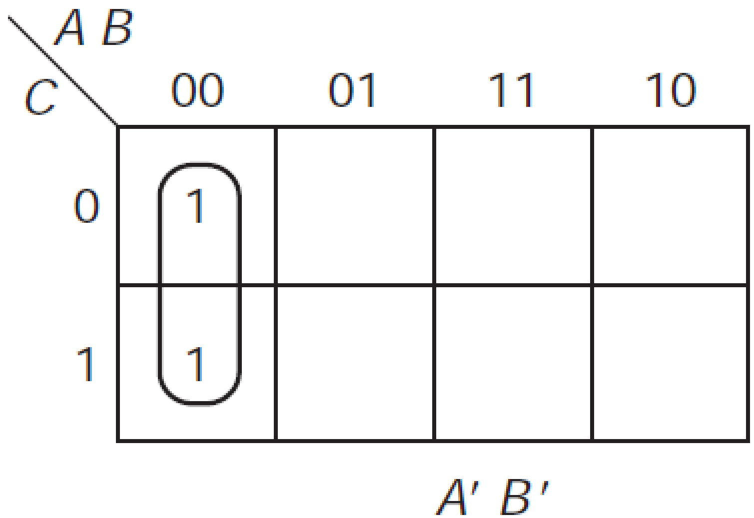
- Map 3.4 is an incorrect K-map because it cannot further combine adjacent minterms into a more compact algebraic format. For exmaple, m_2+m_4 cannot be further simplified.

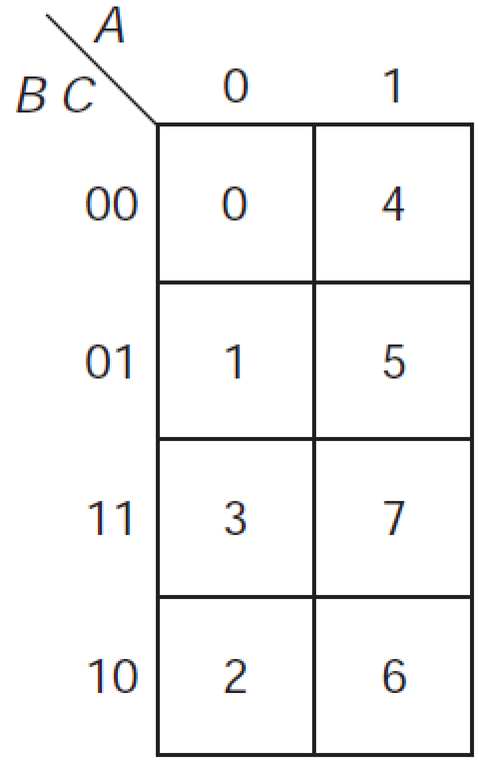
Map 3.4 **Incorrect** arrangement of the map.

C \ AB	00	01	10	11
0	0	2 $A'\,B\,C'$	4 $A\,B'\,C'$	6
1	1	3	5	7

Introduction to the Karnaugh Map (5/13)

- The K-map can be drawn in a vertical orientation. Thus, both the following two maps are feasible and will produce the same results. The key is to let adjacent squares differ by only one bit

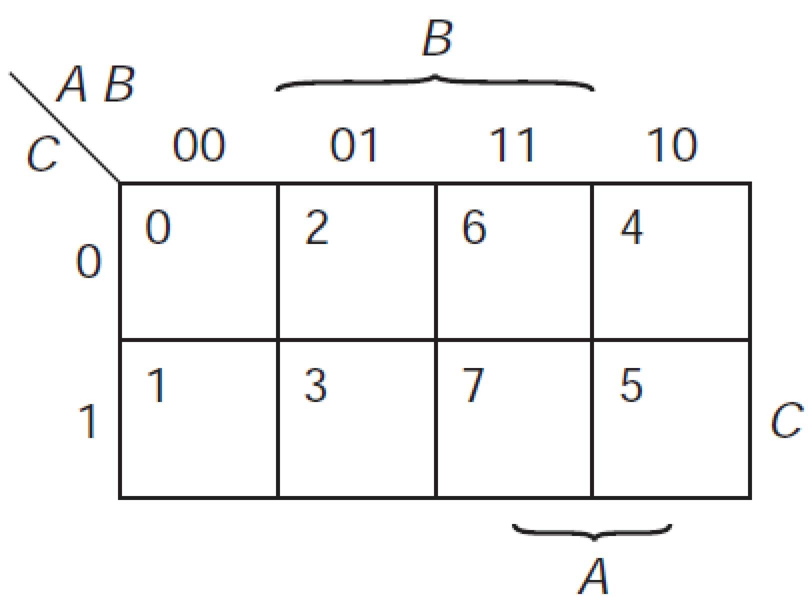




Introduction to the Karnaugh Map (6/13)

- In reading the map, it is useful to label the pairs of columns (in those arrangements where there are four columns) as shown in Map 3.7

Map 3.7 Map with columns labeled.



Introduction to the Karnaugh Map (7/13)

Map 3.8 The four-variable map.

CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

CD \ AB	00	01	11	10
00	$A'B'C'D'$	$A'BC'D'$	$ABC'D'$	$AB'C'D'$
01	$A'B'C'D$	$A'BC'D$	$ABC'D$	$AB'C'D$
11	$A'B'CD$	$A'BCD$	$ABCD$	$AB'CD$
10	$A'B'CD'$	$A'BCD'$	$ABCD'$	$AB'CD'$

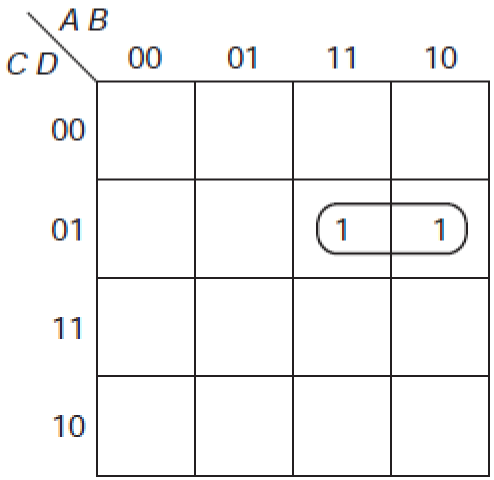
Introduction to the Karnaugh Map (8/13)

- **Example 3.2** Simplify $m_{13}+m_9$, m_3+m_{11}, m_0+m_2 using a K-map.

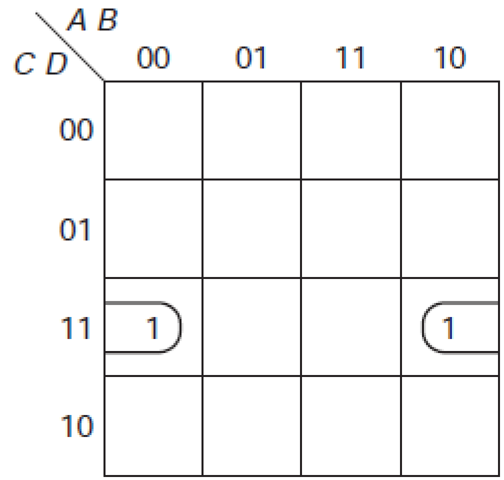
$$m_{13} + m_9: \quad ABC'D + AB'C'D = AC'D$$

$$m_3 + m_{11}: \quad A'B'CD + AB'CD = B'CD$$

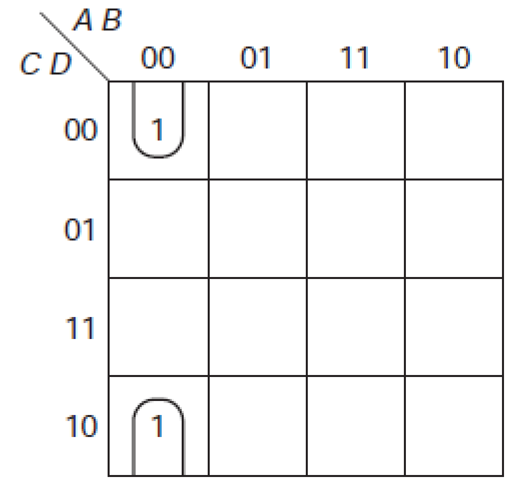
$$m_0 + m_2: \quad A'B'C'D' + A'B'CD' = A'B'D'$$



$AC'D$



$B'CD$



$A'B'D'$

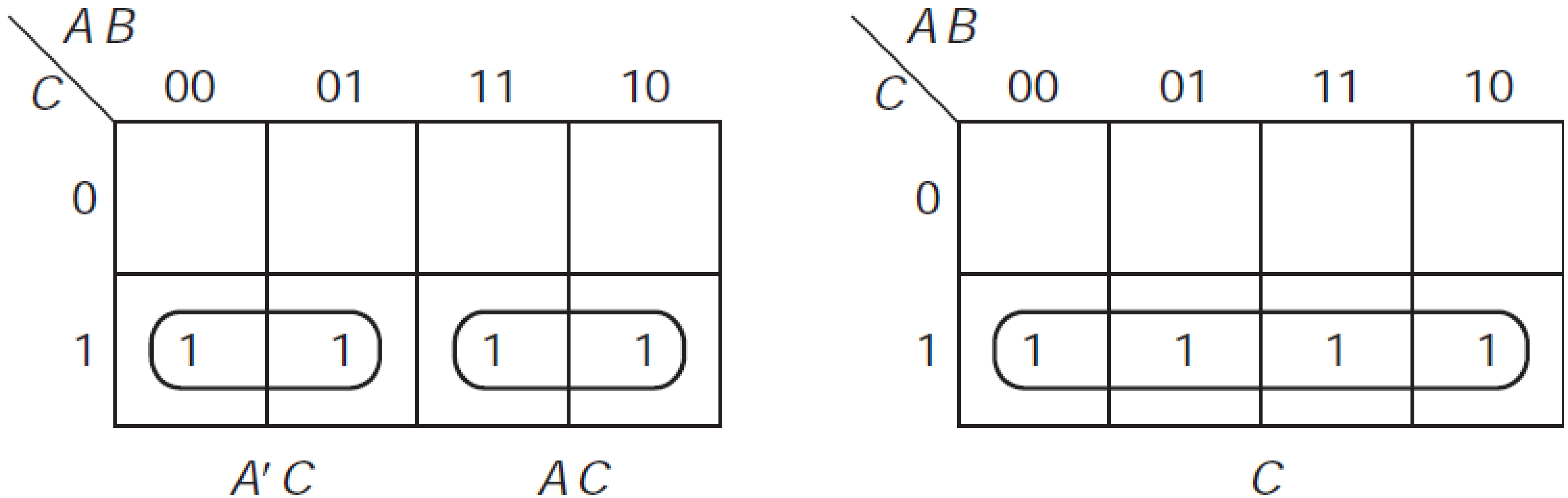
Introduction to the Karnaugh Map (9/13)

- Consider the algebra

$$A'C+AC=C.$$

We can simplify it using a 3-variable K-map

Map 3.9 A group of four 1's.



Introduction to the Karnaugh Map (10/13)

- We can extend P9 to obtain

  P9aa. $a'b' + a'b + ab + ab' = 1$

  P9bb. $(a' + b')(a' + b)(a + b)(a + b') = 0$

- **Proof of P9aa**

  By expanding the equation

  $$(a'+a)(b'+b)=1,$$

  we have

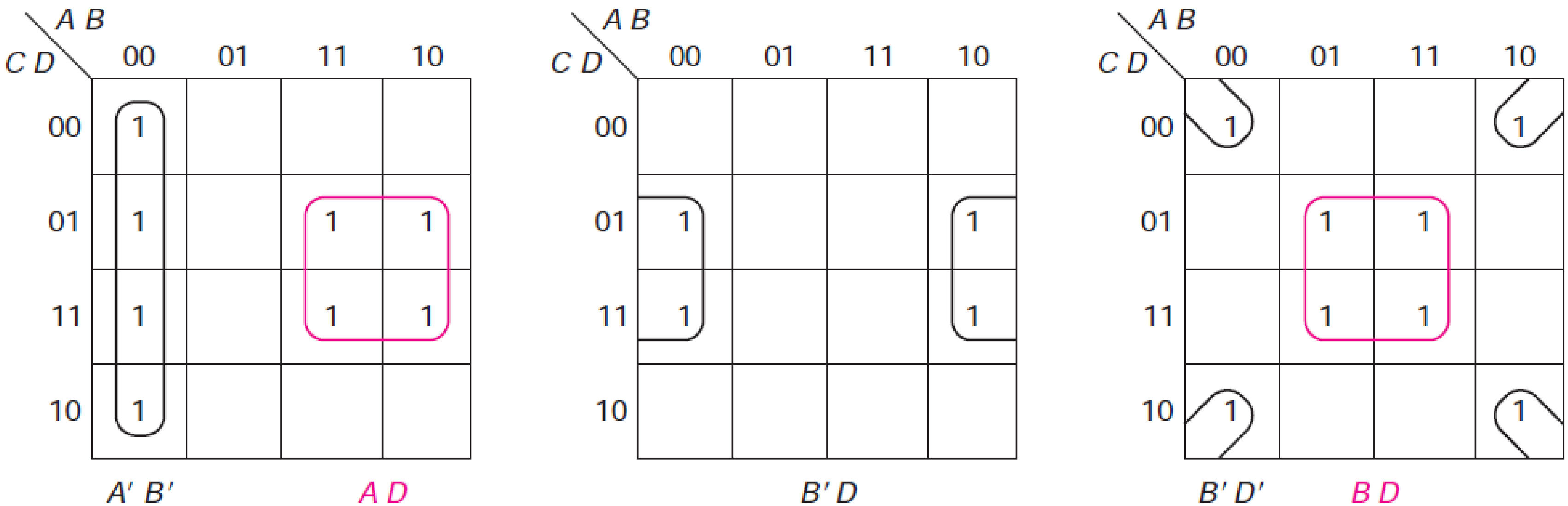
  $$a'b'+a'b+ab+ab'=1.$$

- **Proof of P9bb**

  Complement both sides of P9aa, we get P9bb.

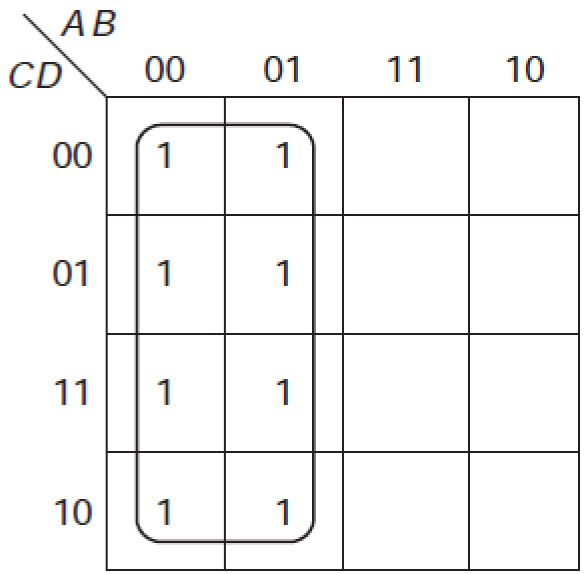
Introduction to the Karnaugh Map (11/13)

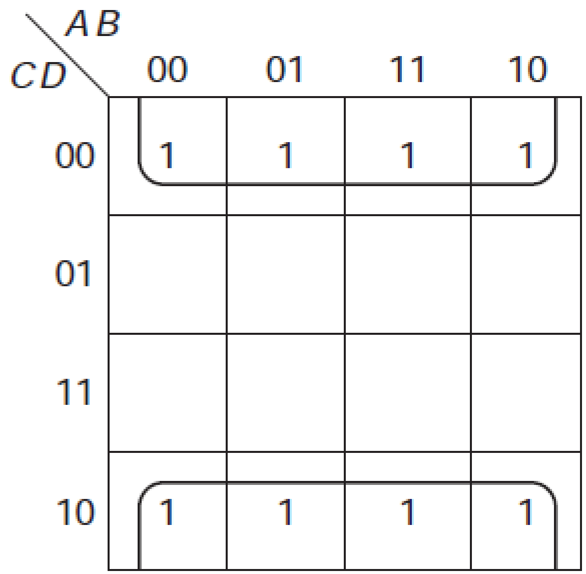
- Some K-map examples of groups of four minterms



Introduction to the Karnaugh Map (12/13)

- Some examples of groups of eight minterms





Introduction to the Karnaugh Map (13/13)

- **Example 3.3.** Map the funtion using a K-map, where

$$F=AB'+AC+A'BC'.$$