Chapter 2 Combinational Systems (III)

A More General Boolean Algebra (1/3)

- The basis for switching algebra is *Boolean algebra*, first published by George Boole in 1849. It allows more than two elements. It is defined in terms of a set of postulates and then the remaining properties are developed from them as theorems
- Some postulates:
  - A Boolean algebra consists of a set of $k \geqq 2$ elements. (For the switching algebra, $k=2$)
  - There are two binary operators, + and $\cdot$, and one unary operator, ’
  - The algebra is closed, that is, if a and b are members of the set, then $a+b$, $a \cdot b$, a' are also members of the set

A More General Boolean Algebra (2/3)

- Some postulates (Cont'd)
  - Commutative law:
    1. $a+b=b+a$
    2. $a \cdot b=b \cdot a$
  - Associative law:
    1. $a+(b+c)=(a+b)+c$
    2. $a \cdot (b \cdot c)=(a \cdot b) \cdot c$
  - Distributive law:
    1. $a+b \cdot c= (a+b) \cdot (a+c)$
    2. $a \cdot (b+c)= a \cdot b+a \cdot c$

A More General Boolean Algebra (3/3)

- Some postulates (Cont'd)
  - Identity:
    1. There exists a unique element in the set, 0, such that $a+0=a$
    2. There exists a unique element in the set, 1, such that $a \cdot 1=a$
  - Complement: For each element a, there exists a unique element a' such that
    1. $a+a'=1$
    2. $a \cdot a'=0$

Table 2.16b Completed definition of OR and AND.

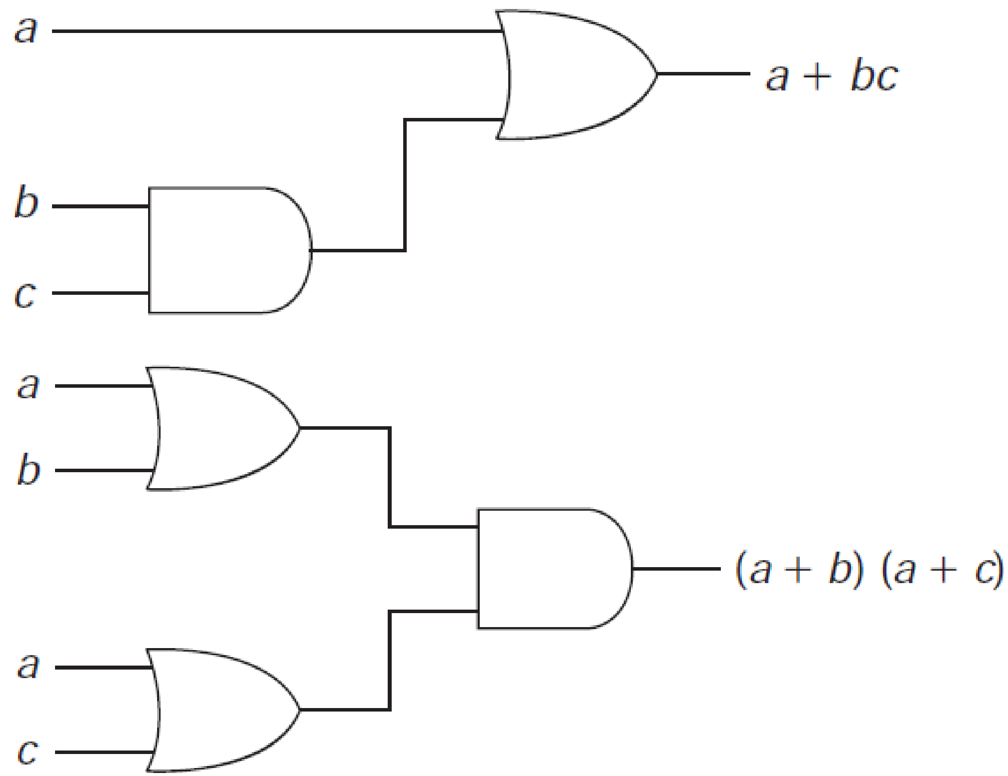
a	b	$a+b$	$a \cdot b$
0	0	0	0
0	1	1	0
1	0	1	0
1	1	1	1

Some Useful Properties of Switching Algebra (1/1)

- P8b. $a+bc=(a+b)(a+c)$
- P9b. $(a+b)(a+b')=a$
- P10a. $a+a'b=a+b$
- P12b. $a(a+b)=a$
- P13a. $at_1+a't_2+t_1t_2=at_1+a't_2$
- P14a. $ab+a'c=(a+c)(a'+b)$

Solved Problems (1/14)

- **Problem 1** Show a block diagram of a circuit using AND and OR gates for each side of P8b: $a+bc=(a+b)(a+c)$
- **Answer 1**



Solved Problems (2/14)

- **Problem 2** Determine whether or not the following expression is equal:

$$f=P'Q'+PR+Q'R$$

$$g=Q'+PQR$$

- **Answer 2**

PQR	$P'Q'$	PR	$Q'R$	f	Q'	PQR	g	
000	1	0	0	1	1	0	1	
001	1	0	1	1	1	0	1	
010	0	0	0	0	0	0	0	
011	0	0	0	0	0	0	0	
100	0	0	0	0	1	0	1	←
101	0	1	1	1	1	0	1	
110	0	0	0	0	0	0	0	
111	0	1	0	1	0	1	1	

The two functions are different.

Solved Problems (3/14)

- **Problem 3** Reduce the following expressions to a minimum SOP form (the number of terms and number of literals in minimum shown in parentheses).

  a. $xyz' + xyz$ (1 term, 2 literals)

  b. $x'y'z' + x'y'z + x'yz + xy'z + xyz$ (2 terms, 3 literals)

  c. $f = abc' + ab'c + a'bc + abc$ (3 terms, 6 literals)

- **Answer 3**

  a. $xyz' + xyz = xy(z' + z) = xy \cdot 1 = xy$ **[P8a, P5aa, P3b]**

  or, in one step, using P9a, where $a = xy$ and $b = z'$

Solved Problems (4/14)

- **Answer 3 (Cont'd)**

b.

Make two copies of $x'y'z$

$$
\begin{aligned}
&= (x'y'z' + x'y'z) + (x'y'z + x'yz) + (xy'z + xyz) && \textbf{[P6a]}\\
&= x'y'(z' + z) + x'z(y' + y) + xz(y' + y) && \textbf{[P8a]}\\
&= x'y' \cdot 1 + x'z \cdot 1 + xz \cdot 1 && \textbf{[P5aa]}\\
&= x'y' + x'z + xz && \textbf{[P3b]}\\
&= x'y' + (x' + x)z = x'y' + 1 \cdot z && \textbf{[P8a, P5aa]}\\
&= x'y' + z && \textbf{[P3bb]}
\end{aligned}
$$

Solved Problems (5/14)

- **Answer 3 (Cont'd)**

c. We make three copies of *abc* and reorganize *f* as follows

$$abc = abc + abc + abc \qquad \textbf{[P6a]}$$

$$f = (abc' + abc) + (ab'c + abc) + (a'bc + abc)$$

$$= ab + ac + bc \qquad \textbf{[P9a]}$$

Solved Problems (6/14)

- **Problem 4** Reduce the following expressions to a minimum POS form (the number of terms and number of literals in minimum shown in parentheses).

  a. $(a+b'+c)(a+b'+c')(a'+b+c)(a'+b'+c)$

  (2 term, 4 literals)

  b. $(x'+y'+z')(x'+y+z')(x'+y+z)$ (2 terms, 4 literals)

- **Answer 4**

  a. We group the first two and the last two terms, and use P9b

  $[(a+b'+c)(a+b'+c')][(a'+b+c)(a'+b'+c)] =$
  $[a+b'][a'+c]$

Solved Problems (7/14)

- **Answer 4 (Cont'd)**

  b. We can make a second copy of the middle term and group it with each of the others

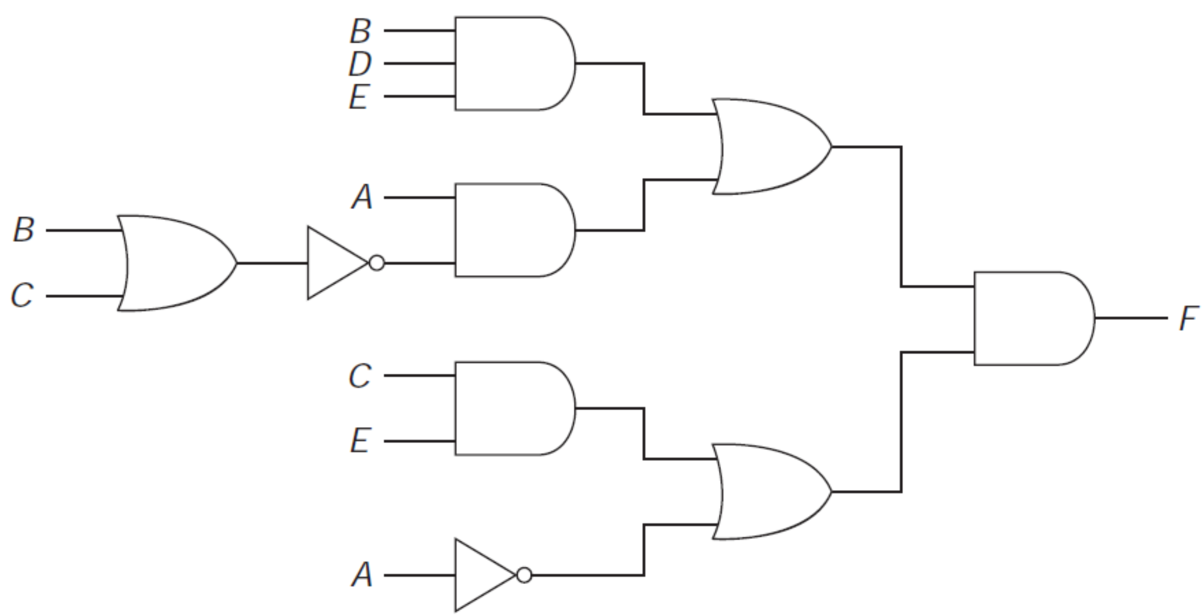
$$(x' + y' + z')(x' + y + z')(x' + y + z)$$
$$= [(x' + y' + z')(x' + y + z')][(x' + y + z)(x' + y + z')]$$
$$= [x' + z'][x' + y]$$

Solved Problems (8/14)

- **Problem 5** Draw a block diagram of a system using AND, OR, and NOT gates to implement the following function. Assume that variables are available only uncomplemented. Do not manipulate the algebra.

  $F=[A(B+C)'+BDE](A'+CE)$

- **Answer 5**



Solved Problems (9/14)

- **Problem 6** For the following function

  $f(x,y,z)=\Sigma m(2,3,5,6,7)$

  a. Show the truth table.

  b. Show an algebraic expression for *f* in sum of minterm form.

  c. Show a minimum SOP expression for *f* (two terms, three literals)

  d. Show the minterms of *f'* in numeric form.

  e. Show an algebraic expression for *f* in product of maxterm form.

  f. Show a minimum POS expression for *f* (two terms, four literals).

Solved Problems (10/14)

- **Answer 6**

a.

$x\,y\,z$	f
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	1
1 1 1	1

b. $f = x'yz' + x'yz + xy'z + xyz' + xyz$

c. $f = x'y + xy'z + xy$

$= y + xy'z$

$= y + xz$

Solved Problems (11/14)

- **Answer 6 (Cont'd)**

d. $f'(x, y, z) = \Sigma m(0, 1, 4)$

e. $f' = x'y'z' + x'y'z + xy'z'$

$f = (x + y + z)(x + y + z')(x' + y + z)$

f. $f' = x'y'z' + x'y'z + xy'z' + x'y'z'$

$= x'y' + y'z'$

$f = (x + y)(y + z)$

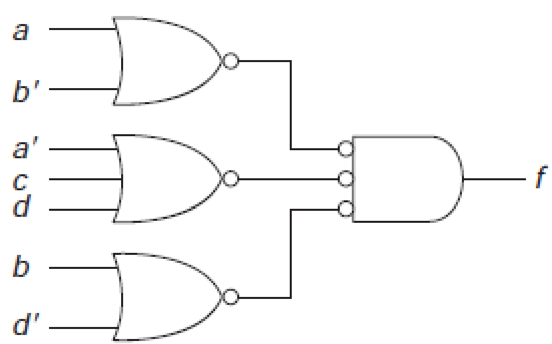
Solved Problems (12/14)

- **Problem 7** Draw a block diagram corresponding to each of the following expressions using only NOR gates. Assume all inputs are available both uncomplemented and ocmplemented. There is no need to manipulate the functions to simplify the algebra

  a. $f=(a+b')(a'+c+d)(b+d')$

  b. $g=[a'b'+a(c+d)](b+d')$

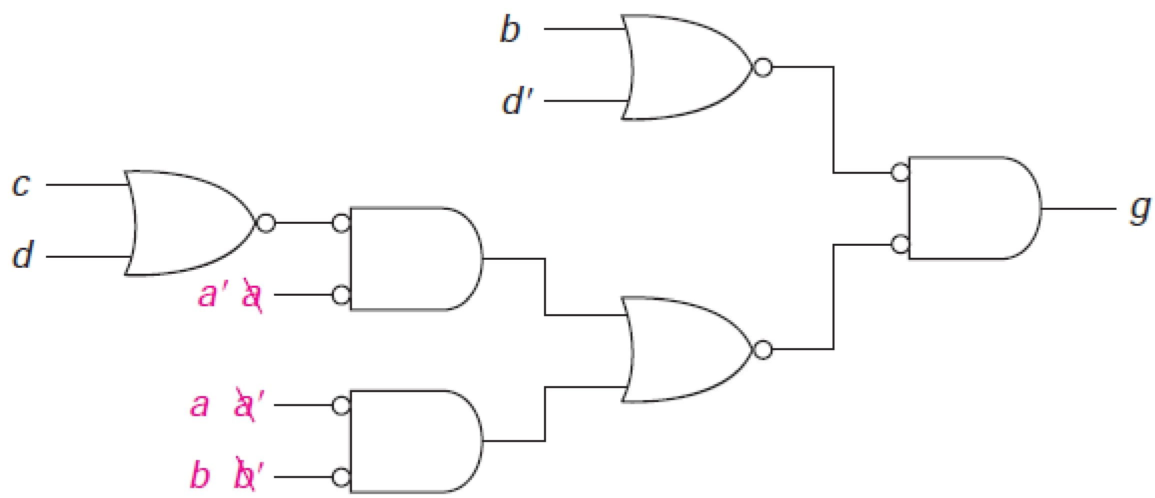
- **Answer 7**



(a)

Solved Problems (13/14)

- **Answer 7 (Cont'd)**



(b)

Solved Problems (14/14)

- **Problem 8** Expand the following function to sum of minterms form

  $F(A,B,C)=A+B'C$

- **Answer 8** We have a choice of two approaches. We could use P3b, P5a (both from right to left) and P8a repeated to produce

$$
\begin{aligned}
A + B'C &= A(B' + B) + (A' + A)B'C \\
&= AB' + AB + A'B'C + AB'C \\
&= AB'(C' + C) + AB(C' + C) + A'B'C + AB'C \\
&= AB'C' + AB'C + ABC' + ABC + A'B'C + AB'C \\
&= AB'C' + AB'C + ABC' + ABC + A'B'C
\end{aligned}
$$