Chapter 6 Effect of Noise on Analog Communication Systems (V)

Effects of Transmission Losses and Noise in Analog Communication Systems (1/2)

- There are two dominant factors that limit the performance of the system. One factor is additive noise that is generated by electronic devices. The second factor is signal attenuation
- If the attenuation factor is $\alpha<1$ and the transmitted signal is $s(t)$, then the received signal is

$$r(t)=\alpha s(t)+n(t)$$

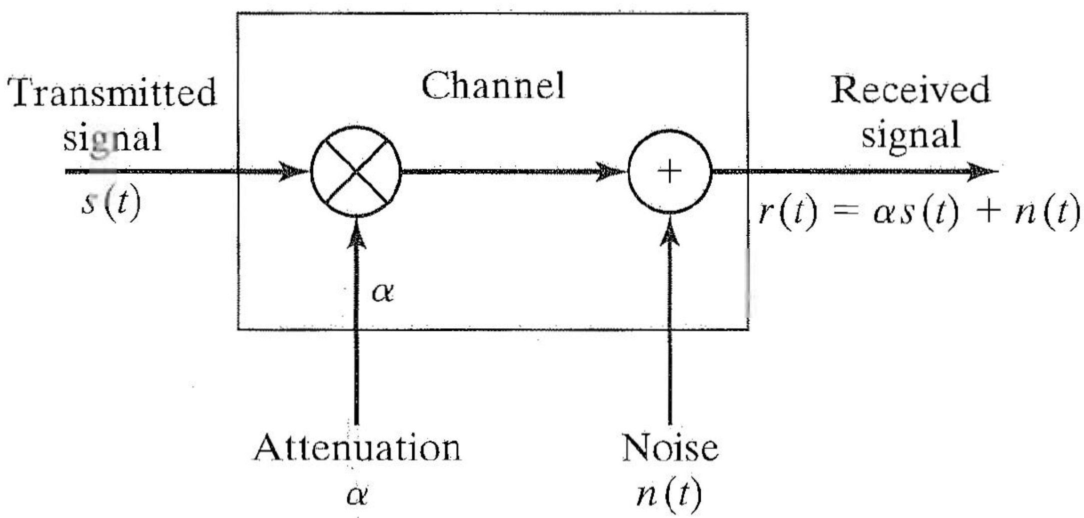

Figure 6.17 Mathematical model of channel with attenuation and additive noise.

Effects of Transmission Losses and Noise in Analog Communication Systems (2/2)

- In many channels, signal attenuation can be offset by using amplifiers to boost the level of the signal during transmission. An amplifier also introduces additive noise in the process of amplification and, thus, corrupts the signal

Characterization of Thermal Noise Sources (1/4)

- Any conductive two-terminal device is generally characterized as lossy and has some resistance
- A resistor that is at a temperature *T* above absolute zero contains free electrons that exhibit random motion and, thus, result in a noise voltage across the terminals of the resistor. Such a noise voltage is called thermal noise

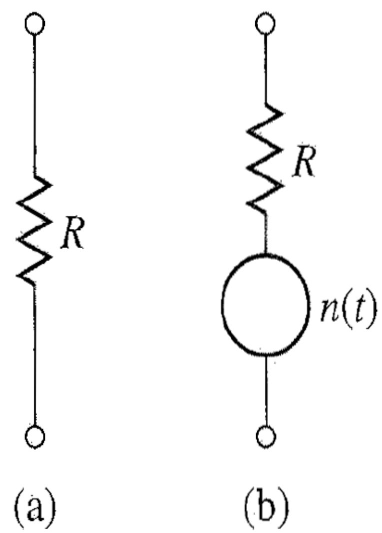

Figure 6.18 A physical resistor (a) is modeled as a noiseless resistor in series with a noise source (b).

Characterization of Thermal Noise Sources (2/4)

- The power spectral density of thermal noise is given as

$$S_R(f) = \frac{2R\hbar|f|}{(e^{\frac{\hbar|f|}{kT}} - 1)} \ (volts)^2 / Hz$$

$\hbar$ is Plank's constant, k is Boltzmann's constant, and T is the temperature of the resistor in degrees Kelvin

- At frequencies below 10^{12} Hz

$$e^{\frac{\hbar|f|}{kT}} \approx 1 + \frac{\hbar|f|}{kT}$$

Consequently the power spectral density is well approximated as

$$S_R(f) = 2RkT \ (volt)^2 / Hz$$

Characterization of Thermal Noise Sources (3/4)

- When connected to a load resistance with value R_L, the noise voltage shown in Fig. 6.19 delivers the maximum power when $R=R_L$
- The maximum power delivered to the load is

$$P_L = i^2 R_L = \left(\frac{n(t)}{2R_L}\right)^2 R_L = \frac{n^2(t)}{4R_L}$$

$$E[P_L] = \frac{E[n^2(t)]}{4R_L}$$

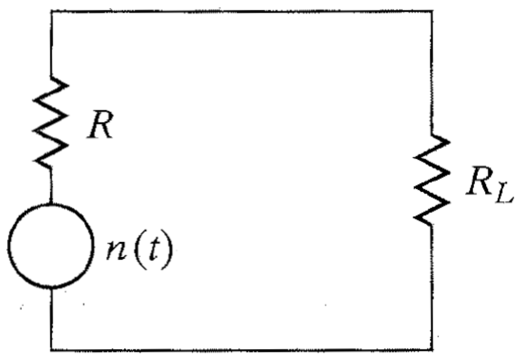

Figure 6.19 Noisy resistor connected to a load resistance R_L.

Characterization of Thermal Noise Sources (4/4)

- We may divide $S_R(f)$ by $4R$ to obtain the power spectral density. The power spectral density of the noise voltage across the load resistor is

$$S_n(f) = \frac{kT}{2} \ W/Hz$$

  kT is usually denoted by N_0. The power spectral density of thermal noise is expressed as

$$S_n(f) = \frac{N_0}{2} \ W/Hz$$

- At room temperature (T=290 K), N_0=4×10^{-21} W/Hz

Effective Noise Temperature and Noise Figure (1/7)

- Any amplifier has some finite passband, we may model an amplifier as a filter with the frequency response characteristic $H(f)$
- Consider Fig. 6.20. We recall that the noise power at the output of the amplifier is

$$P_{no} = \int_{-\infty}^{\infty} S_n(f)\,|H(f)|^2\,df = \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\,df$$

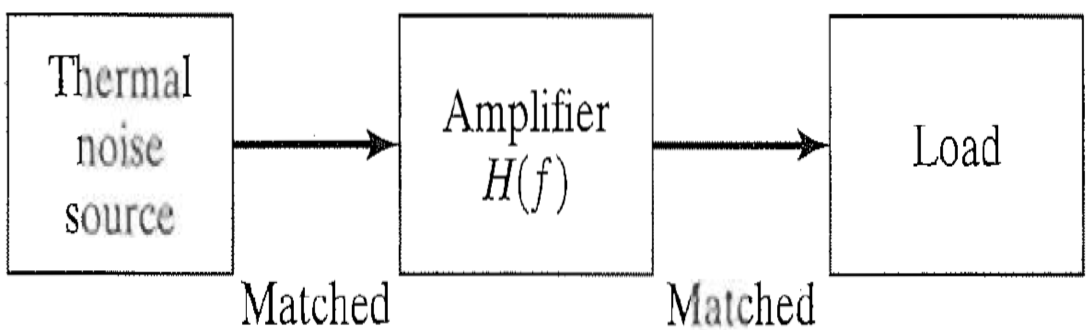

Figure 6.20 Thermal noise connected to amplifier and load.

Effective Noise Temperature and Noise Figure (2/7)

- Recall that the noise equivalent bandwidth of the filter is defined as

$$B_{neq} = \frac{1}{2G}\int_{-\infty}^{\infty} |H(f)|^2 \, df$$

By definition, $G = |H(f)|_{max}^{2}$ is the maximum available power gain of the amplifier

- The output noise power from an ideal amplifier that introduces no additional noise may be expressed as

$$P_{no} = GN_0B_{neq}$$

- Any practical amplifier introduces additional noise at its output due to internally generated noise. We have

$$\begin{aligned} P_{no} &= GN_0B_{neq} + P_{ni} \\ &= GkTB_{neq} + P_{ni} \end{aligned}$$

Effective Noise Temperature and Noise Figure (3/7)

- Therefore,

$$P_{no} = GkB_{neq}\left(T + \frac{P_{ni}}{GkB_{neq}}\right)$$

This leads us to define a quantity

$$T_e = \frac{P_{ni}}{GkB_{neq}}$$

T_e is called the effective noise temperature of the two-port network (amplifier). Then

$$P_{no} = GkB_{neq}\left(T + T_e\right)$$

- We interpret the output noise as originating from a thermal noise source at the temperature $T+T_e$

Effective Noise Temperature and Noise Figure (4/7)

- If the power spectral density of a signal source is constant, a signal source at the input to the amplifier with power P_{si} will produce an output with power

$$P_{so} = GP_{si}$$

- The output SNR from the two-port network is

$$\left(\frac{S}{N}\right)_o = \frac{P_{so}}{P_{no}} = \frac{GP_{si}}{GkTB_{neq}(1+T_e/T)}$$
$$= \frac{P_{si}}{N_0 B_{neq}(1+T_e/T)}$$
$$= \frac{1}{1+T_e/T}\left(\frac{S}{N}\right)_i \qquad (6.5.15)$$

- The output SNR is degraded (reduced) by the factor $(1+T_e/T)$. T_e is a measure of the noisiness of the amplifier

Effective Noise Temperature and Noise Figure (5/7)

- The ratio

$$F = \left(1 + \frac{T_e}{T}\right)$$

with T=290 K is called the noise figure of the amplifier

- Eq. (6.5.15) may be expressed as

$$\left(\frac{S}{N}\right)_o = \frac{1}{F}\left(\frac{S}{N}\right)_i \qquad (6.5.17)$$

- By taking the logarithm on both sides of Eq. (6.5.17), we obtain

$$10\log_{10}\left(\frac{S}{N}\right)_o = -10\log_{10} F + 10\log_{10}\left(\frac{S}{N}\right)_i$$

$10\log_{10}F$ represents the loss in SNR due to the additional noise introduced by the amplifier

- The noise figure for amplifiers is around 3 dB to 7 dB

Effective Noise Temperature and Noise Figure (6/7)

- Consider a cascade of two amplifiers with gains G_k and corresponding noise figures F_k, $1 \leqq k \leqq 2$. We can relate

$$G_1 G_2 k T_e B_{neq} = G_1 G_2 k T_{e1} B_{neq} + G_2 k T_{e2} B_{neq.}$$

Thus,

$$T_e = T_{e1} + \frac{T_{e2}}{G_1} \qquad (*)$$

Substituting $F = 1 + T_e / T$ into (*), we obtain

$$F = F_1 + \frac{F_2 - 1}{G_1}$$

- For K amplifiers, we have the *Fries' formula*

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_K - 1}{G_1 G_2 \cdots G_{K-1}}. \qquad (6.5.19)$$

The dominant term is F_1, which is the noise figure of the first amplifier stage

Effective Noise Temperature and Noise Figure (7/7)

- The front end of a receiver should have a low noise figure and a high gain. In that case, the remaining terms in the sum will be negligible
- **Example 6.5.1.** Suppose an amplifier is designed with three identical stages, each of which has a gain of G_k=5 and a noise figure F_i=6, i=1,2,3. Determine the overall noise figure of the cascade of the three stages.
- From Eq. (6.5.19), we obtain

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$$

Hence,

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} = 7.2$$

or, equivalently, F=8.57 dB

Transmission Losses (1/5)

- Any physical channel attenuates the signal transmitted through it
- The amount of signal attenuation generally depends on the physical medium, the frequency of operation, and the distance between the transmitter and the receiver
- We define the loss L in signal transmission as the ratio of the input (transmitted) power to the output (received) power of the channel, *i.e.*,

$$L = \frac{P_T}{P_R}$$

- In decibels,

$$L_{dB} = 10\log_{10} L = 10\log_{10} P_T - 10\log_{10} P_R$$

Transmission Losses (2/5)

- In wireline channels, the transmission loss is usually given in terms of decibels per unit length, *e.g.*, dB/km. For example, the transmission loss in coaxial cable of 1 cm diameter is about 2 dB/km at a frequency of 1 MHz. The loss generally increases with an increase in frequency
- **Example 6.5.2.** Determine the transmission loss for a 10 km and a 20 km coaxial cable if the loss per kilometer is 2 dB at the frequency of operation
- The loss for the 10 km channel is L=20 dB. For the 20 km channel, the loss is L=40 dB

Transmission Losses (3/5)

- In line-of-sight radio systems, the transmission loss is given as

$$L = \left(\frac{4\pi d}{\lambda}\right)^2 \qquad (6.5.22)$$

where $\lambda = c/f$ is the wavelength of the transmitted signal, c is the speed of light (3×10^8 m/s), f is the frequency of the transmitted signal, and d is the distance between the transmitter and the receiver in meters

- In radio transmission, L is called the *free-space path loss*

Transmission Losses (4/5)

- **Example 6.5.3.** Determine the free-space path loss for a signal transmitted at f=1 MHz over distance of 10 km and 20km.
- The loss given in Eq. (6.5.22) for a signal at a wavelength λ=300 m is

$$L_{dB} = 10\log_{10} L = 20\log_{10}(4\pi \times 10^4 / 300)$$
$$= 52.44\ dB$$

for the 10 km path and

$$L_{dB} = 10\log_{10} L = 20\log_{10}(8\pi \times 10^4 / 300)$$
$$= 58.44\ dB$$

for the 20 km path

- It is interesting to note that doubling the distance in radio transmission increases the free-space path loss by 6 dB

Transmission Losses (5/5)

- **Example 6.5.4.** A signal is transmitted through a 10 km coaxial line channel, which exhibits a loss of 2 dB/km. The transmitted signal power is P_{dB}=-30 dBW (-30 dBW means 30 dB below 1 watt or simply, one milliwatt). Determine the received signal power and the power at the output of an amplifier (at the receiver side) that has a gain of 15 dB.
- The transmission loss for the 10 km channel is L=20 dB. Hence, the receiver signal power is

$$P_{RdB}=-30-20=-50 \text{ dBW}$$

- The amplifier boosts the received signal power by 15 dB. Hence, the power at the output of the amplifier is

$$P_{odB}=-50+15=-35 \text{ dBW}$$

Repeaters for Signal Transmission (1/9)

- Analog repeaters are generally amplifiers that are generally used in telephone wireline channels and microwave line-of-sight radio channels to boost the signal level and, thus, to offset the effect of signal attenuation in transmission through the channel
- The input signal at the input to the repeater is

$$P_R = P_T / L$$

- Assume the repeater gain is G. The output power from the repeater is

$$P_o = G P_R = G P_T / L$$

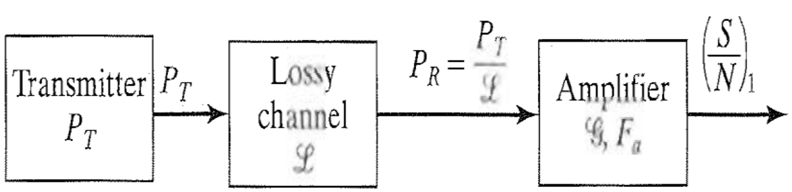

Figure 6.21 A communication system employing a repeater to compensate for channel loss.

Repeaters for Signal Transmission (2/9)

- We may select the amplifier gain G to offset the transmission loss. Hence $G=L$ and $P_o=P_T$
- The SNR at the output of the repeater is

$$\left(\frac{S}{N}\right)_1 = \frac{1}{F_a}\left(\frac{S}{N}\right)_i$$
$$= \frac{1}{F_a}\left(\frac{P_R}{N_0 B_{neq}}\right) = \frac{1}{F_a}\left(\frac{P_T}{L N_0 B_{neq}}\right)$$
$$= \frac{1}{F_a L}\left(\frac{P_T}{N_0 B_{neq}}\right)$$

- $(S/N)_i$ is defined as the SNR at the amplifier input
- We may view the lossy transmission medium followed by the amplifier as a cascade of two networks: one with a noise figure L and the other with a noise figure F_a

Repeaters for Signal Transmission (3/9)

- For the cascade connection, the overall noise figure is

$$F = L + \frac{F_a - 1}{G'}$$

If we select $G' = 1/L$, then

$$F = L + \frac{F_a - 1}{1/L} = LF_a$$

Hence, the cascade of the lossy transmission medium and the amplifier is equivalent to a single network with the noise figure LF_a

Repeaters for Signal Transmission (4/9)

- Suppose that we transmit the signal over *K* segments of the channel, where each segment has its own repeater, as shown in Fig. 6.22

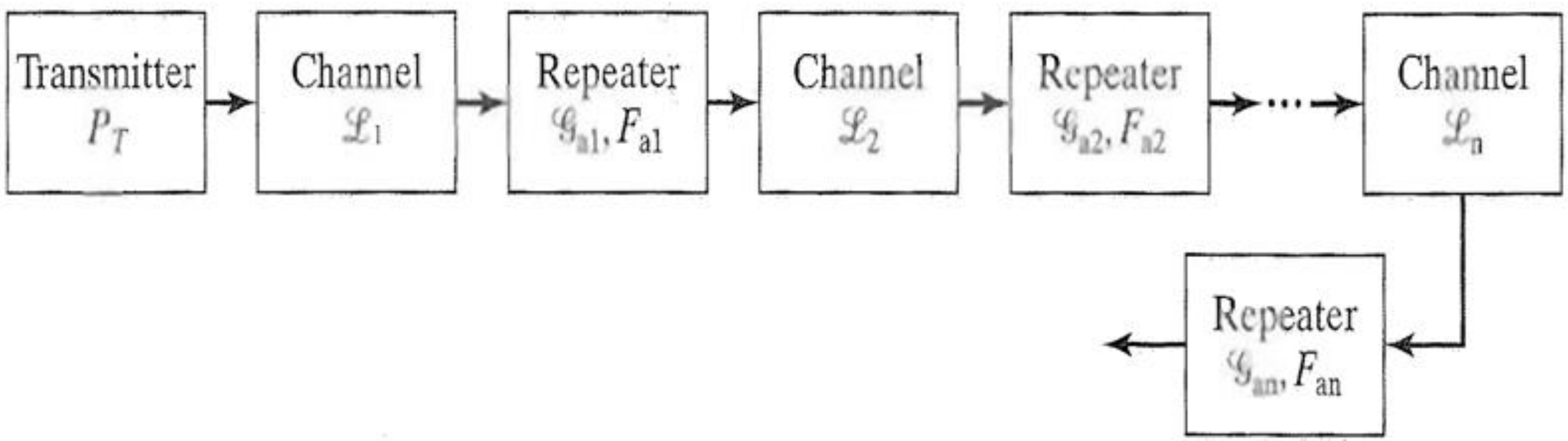

Figure 6.22 A communication system employing repeaters.

Repeaters for Signal Transmission (5/9)

- If $F_i = LF_{ai}$ is the noise figure of the ith section, the overall noise figure for the K sections is

$$F = L_1 F_{a1} + \frac{L_2 F_{a2} - 1}{G_{a1}/L_1} + \frac{L_3 F_{a3} - 1}{(G_{a1}/L_1)(G_{a2}/L_2)} + \cdots + \frac{L_K F_{aK} - 1}{(G_{a1}/L_1)(G_{a2}/L_2)\cdots(G_{aK-1}/L_{K-1})}$$

- The signal-to-noise ratio at the output of the repeater is

$$\left(\frac{S}{N}\right)_o = \frac{1}{F}\left(\frac{S}{N}\right)_i = \frac{1}{F}\left(\frac{P_T}{N_0 B_{neq}}\right)$$

- $(S/N)_i$ is defined as the SNR at the transmitter output

Repeaters for Signal Transmission (6/9)

- In the important special case where K segments are identical, i.e., $L_i=L$ for all i and $F_{ai}=F_a$ for all i, and $G_{ai}=L_i$, then the overall noise figure becomes

$$F=KLF_a-(K-1)\doteqdot KLF_a$$

- Hence,

$$\left(\frac{S}{N}\right)_o \approx \frac{1}{KLF_a}\left(\frac{P_T}{N_0 B_{neq}}\right)_i . \qquad (6.5.35)$$

Therefore, the overall noise figure for the cascade of the K identical segment is simply K times the noise figure of one segment

Repeaters for Signal Transmission (7/9)

- **Example 6.5.5.** A signal with the bandwidth 4 kHz is to be transmitted a distance of 200 km over a wireline channel that has an attenuation of 2 dB/km. (a) Determine the transmitter power P_T required to achieve an SNR of $(S/N)_o=30$ dB at the output of the receive amplifier that has a noise figure $F_a=5$ dB. (b) Repeat the calculation when a repeater is inserted every 10 km in the wireline channel, where the repeater has a gain of 20 dB and a noise figure of $F_a=5$ dB. Assume that the noise equivalent bandwidth of each repeater is $B_{neq}=4$ kHz and that $N_0=4\times10^{-21}$ W/Hz

Repeaters for Signal Transmission (8/9)

- **Example 6.5.5. (Cont'd)**
- (a) From Eq. (6.5.35) with $K=1$, we have

$$10\log_{10}(S/N)_o = -10\log_{10} L - 10\log_{10} F_a - 10\log_{10}(N_0 B_{neq}) + 10\log_{10}(P_T)$$

Hence,

$$P_{TdB} = (S/N)_{\mathrm{odB}} + F_{adB} + (N_0 B_{neq})_{dB} + 10\log_{10}(L)$$
$$= 267 \text{ dB}$$
$$P_T = 5\times 10^{26} \text{ Watts},$$

which is an astronomical figure

- (b) The use of a repeater every 10 km reduces the loss per segment to $L=20$ dB. There are 20 repeaters and each repeater has a noise figure of 5 dB

Repeaters for Signal Transmission (9/9)

- **Example 6.5.5. (Cont'd)**
- Hence, Eq. (6.5.35) yields

$$P_{TdB} = -100 \text{ dBW}$$

or equivalently,

$$P_T = 10^{-10} \text{ Watts}$$

- From Eq. (6.5.35) that the transmitted power P_T must be increased linearly with the number K of repeaters in order to maintain the $(S/N)_o$ as K increases
- For every factor of two increase in K, the transmitted power P_T must be increased by 3 dB