Chapter 6 Effect of Noise on Analog Communication Systems (I)

Introduction (1/1)

- In this chapter, the effect of noise on various analog communication systems will be analyzed
- Angle modulation systems and FM can provide a high degree of noise immunity; therefore, they are desirable in cases of severe noise and/or low signal power
- This noise immunity is obtained at the price of sacrificing channel bandwidth
- We determine the signal-to-noise ratio (SNR) of the output of receiver for various analog modulations

Effect of Noise on Baseband System (1/2)

- Since baseband systems serve as a basis for comparison of various modulation systems, we begin with a noise analysis of a baseband system
- In baseband systems, there is no carrier demodulation to be performed. The receiver consists only of an ideal lowpass filter with the bandwidth W
- The noise power at the output of the receiver, for a white noise input is

$$P_{n_0} = \int_{-W}^{+W} \frac{N_0}{2} df$$
$$= N_0 W$$

Effect of Noise on Baseband System (2/2)

- If we denote the received power by P_R, the baseband SNR is given by

$$\left(\frac{S}{N}\right)_b = \frac{P_R}{N_0 W}$$

- **Example 6.1.1.** Find the SNR in a baseband system with a bandwidth of 5 kHz and with $N_0 = 10^{-14}$ W/Hz. The transmitter power is one kilowatt and the channel attenuation is 10^{-12}

- We have $P_R = 10^{-12} P_T = 10^{-12} \times 10^3 = 10^{-9}$ Watts. Therefore,

$$\left(\frac{S}{N}\right)_b = \frac{P_R}{N_0 W} = \frac{10^{-9}}{10^{-14} \times 5000} = 20$$

This is equivalent to $10\log_{10}20 = 13$ dB

Effect of Noise on DSB-SC AM (1/5)

- In DSB-SC AM, the transmitted signal is

$$u(t) = A_c m(t)\cos(2\pi f_c t)$$

The received signal at the output of the receiver filter is the sum of this signal and filtered noise

- Adding the filtered noise to the modulated signal, we can express the received signal as

$$\begin{aligned} r(t) &= u(t) + n(t) \\ &= A_c m(t)\cos(2\pi f_c t) \\ &\quad + n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t) \end{aligned}$$

Effect of Noise on DSB-SC AM (2/5)

- We demodulate the received signal by first multiplying $r(t)$ by a locally generated sinusoid $\cos(2\pi f_c t+\varphi)$ and then passing the product signal through an ideal lowpass filter having a bandwidth W
- The multiplication of $r(t)$ with $\cos(2\pi f_c t+\varphi)$ yields

$$
\begin{aligned}
r(t)\cos(2\pi f_c t+\phi) &= A_c m(t)\cos(2\pi f_c t)\cos(2\pi f_c t+\phi) \\
&\quad + n(t)\cos(2\pi f_c t+\phi) \\
&= \tfrac{1}{2}A_c m(t)\cos(\phi)+\tfrac{1}{2}A_c m(t)\cos(4\pi f_c t+\phi) \\
&\quad + \tfrac{1}{2}[n_c(t)\cos(\phi)+n_s(t)\sin(\phi)] \\
&\quad + \tfrac{1}{2}[n_c(t)\cos(4\pi f_c t+\phi)-n_s(t)\sin(4\pi f_c t+\phi)]
\end{aligned}
$$

Effect of Noise on DSB-SC AM (3/5)

- The lowpass filter passes only the lowpass components. Its output is

$$y(t) = \tfrac{1}{2} A_c m(t)\cos(\phi) + \tfrac{1}{2}[n_c(t)\cos(\phi) + n_s(t)\sin(\phi)]$$

- The effect of a phase-locked loop is to generate a sinusoidal carrier at the receiver with the same frequency and phase of the received carrier
- If a phase-locked loop is employed, then $\varphi=0$ and the demodulator is called a *coherent* or *synchronous demodulator*
- Assume we employ a coherent demodulator; hence

$$y(t) = \tfrac{1}{2}[A_c m(t) + n_c(t)]$$

Effect of Noise on DSB-SC AM (4/5)

- The message signal power is given by

$$P_o = \frac{A_c^2}{4} P_M$$

P_M is the power content of the message signal. The noise power is given by

$$P_{n_0} = \frac{1}{4} P_{n_c}$$

$$= \frac{1}{4} P_n,$$

- The power spectral density of $n(t)$ is given by

$$S_n(f) = \begin{cases} \frac{N_0}{2}, & |f \pm f_c| \leq W \\ 0, & otherwise \end{cases}$$

Effect of Noise on DSB-SC AM (5/5)

- The noise power is

$$P_n = \int_{-\infty}^{\infty} S_n(f)df = \frac{N_0}{2} \times 4W = 2WN_0$$

- The output SNR is

$$\left(\frac{S}{N}\right)_o = \frac{P_o}{P_{n_o}} = \frac{\frac{A_c^2}{4} P_M}{\frac{1}{4} 2WN_0} = \frac{A_c^2 P_M}{2WN_0} \qquad (6.1.2)$$

The received signal power is $A_c{}^2 P_M / 2$. Therefore, the output SNR in Eq. (6.1.2) may be expressed as

$$\left(\frac{S}{N}\right)_o = \frac{P_R}{N_0 W}$$

- In DSB-SC AM, the output SNR is the same as the SNR for a baseband system. In other words, DSB-SC AM does not provide any SNR improvement over a baseband communication system

Effect of Noise on SSB AM (1/3)

- The modulated signal is given by

$$u(t) = A_c m(t)\cos(2\pi f_c t) \pm A_c \hat{m}(t)\sin(2\pi f_c t)$$

- The input to the demodulator is

$$u(t) = A_c m(t)\cos(2\pi f_c t) \pm A_c \hat{m}(t)\sin(2\pi f_c t) + n(t)$$

$$= (A_c m(t) + n_c(t))\cos(2\pi f_c t) + (\pm A_c \hat{m}(t) - n_s(t))\sin(2\pi f_c t)$$

- Assume that demodulation occurs with an ideal phase reference. The output of the lowpass filter is the in-phase component. That is,

$$y(t) = \frac{A_c}{2} m(t) + \frac{1}{2} n_c(t)$$

Effect of Noise on SSB AM (2/3)

- We have

$$P_O = \frac{A_c^2}{4} P_M$$

and

$$P_{n_o} = \frac{1}{4} P_{n_c} = \frac{1}{4} P_n$$

where

$$P_n = \int_{-\infty}^{\infty} S_n(f)df = \frac{N_0}{2} \times 2W = WN_0$$

- Therefore,

$$\left(\frac{S}{N}\right)_o = \frac{P_o}{P_{n_o}} = \frac{A_c^2 P_M}{WN_0}$$

Effect of Noise on SSB AM (3/3)

- We can relate the received power P_R as follows

$$P_R = A_c^2 P_M$$

thus,

$$\left(\frac{S}{N}\right)_{oSSB} = \frac{P_R}{WN_0} = \left(\frac{S}{N}\right)_b$$

- The signal-to-noise ratio in an SSB system is equivalent to that of a DSB system

Effect of Noise on Conventional AM (1/12)

- In conventional DSB AM, the modulated signal is given as

$$u(t) = A_c[1 + am_n(t)]\cos(2\pi f_c t)$$

- The received signal at the input to the demodulator is

$$r(t) = A_c[1 + am_n(t) + n_c(t)]\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

where a is the modulation index and $m_n(t)$ is normalized so that its minimum value is -1

- If a synchronous demodulator is employed, the situation is basically similar to the DSB case. After mixing and lowpass filtering, we have

$$y_l(t) = \tfrac{1}{2}[A_c(1 + am_n(t)) + n_c(t)]$$

Effect of Noise on Conventional AM (2/12)

- The desired signal is $m_n(t)$. The DC component is removed by a DC block and the lowpass filtered output is

$$y(t) = \tfrac{1}{2} A_c a m_n(t) + \tfrac{1}{2} n_c(t)$$

- The received signal power is

$$P_R = \frac{A_c^{\,2}}{2}[1 + a^2 P_{M_n}]$$

where we have assumed that the message process is zero mean

Effect of Noise on Conventional AM (3/12)

- The output SNR calculated using $y(t)$ is

$$\left(\frac{S}{N}\right)_{oAM} = \frac{\frac{1}{4}A_c^2 a^2 P_{M_n}}{\frac{1}{4}P_{n_c}}$$

$$= \frac{A_c^2 a^2 P_{M_n}}{2N_0 W}$$

$$= \frac{a^2 P_{M_n}}{1+a^2 P_{M_n}} \frac{\frac{A_c^2}{2}[1+a^2 P_{M_n}]}{N_0 W}$$

$$= \frac{a^2 P_{M_n}}{1+a^2 P_{M_n}} \frac{P_R}{N_0 W}$$

$$= \frac{a^2 P_{M_n}}{1+a^2 P_{M_n}} \left(\frac{S}{N}\right)_b$$

$$= \eta \left(\frac{S}{N}\right)_b$$

- η denotes the modulation efficiency

Effect of Noise on Conventional AM (4/12)

- Since $\eta < 1$, the SNR in conventional AM is always smaller than the SNR in a baseband system
- For a practical speech signal, the overall loss in SNR when compared to a baseband signal is around 11 dB. The reason for this loss is that a large part of the transmitter power is used to send the carrier component of the modulated signal and not the desired signal
- The input to the envelope detector is

$$r(t) = [A_c(1 + am_n(t)) + n_c(t)]\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t);$$

the envelope of $r(t)$ is

$$V_r(t) = \sqrt{[A_c(1 + am_n(t)) + n_c(t)]^2 + n_s^{\,2}(t)}$$

Effect of Noise on Conventional AM (5/12)

- Assume the signal component in $r(t)$ is much stronger than the noise component; in other words,

$$\Pr(n_c(t) << A_c[1+am_n(t)]) \approx 1$$

- We have a high probability that

$$V_r(t) \approx A_c[1+am_n(t)]+n_c(t)$$

After removing the DC component, we obtain

$$y(t) = A_c am_n(t) + n_c(t)$$

- $y(t)$ is basically the same as that of the synchronous DSB-SC/SSB demodulation without the ½ coefficient. This coefficient has no effect on the final SNR

Effect of Noise on Conventional AM (6/12)

- We conclude that under the assumption of high SNR at the receiver input, the performance of synchronous and envelope demodulators is the same
- When the noise power is much stronger than the signal power, we have

$$
\begin{aligned}
V_r(t) &= \sqrt{(A_c(1+am_n(t))+n_c(t))^2+n_s^{\,2}(t)} \\
&= \sqrt{A_c^{\,2}(1+am_n(t))^2+n_c^{\,2}(t)+n_s^{\,2}(t)+2A_cn_c(t)(1+am_n(t))} \\
&\overset{a}{\approx} \sqrt{(n_c^{\,2}(t)+n_s^{\,2}(t))[1+\tfrac{2A_cn_c(t)}{n_c^{\,2}(t)+n_s^{\,2}(t)}(1+am_n(t))]} \\
&\overset{b}{\approx} V_n(t)\left[1+\tfrac{A_cn_c(t)}{V_n^{\,2}(t)}(1+am_n(t))\right] \\
&= V_n(t)+\tfrac{A_cn_c(t)}{V_n(t)}(1+am_n(t))
\end{aligned}
$$

Effect of Noise on Conventional AM (7/12)

- (a) uses the fact that $A_c^{\ 2}(1+am_n(t))^2$ is small compared with the other components
- (b) denotes $\sqrt{n_c^{\ 2}(t)+n_s^{\ 2}(t)}$ by $V_n(t)$ and uses the approximation $\sqrt{1+\varepsilon}\approx 1+\frac{\varepsilon}{2}$
- At the demodulator output, the signal and the noise components are no longer additive. The signal component is multiplied by noise and is no longer distinguishable. In this case, no meaningful SNR can be defined. We say that the system is operating below the threshold

Effect of Noise on Conventional AM (8/12)

- **Example 6.1.2.** We assume the message $M(t)$ is a WSS process with the autocorrelation function

$$R_M(\tau) = 16\sin c^2(10{,}000\tau)$$

and $\max|m(t)| = 6$. The channel has a 50 dB attenuation and an additive white noise with the PSD $S_n(f) = N_0/2 = 10^{-12}$ Watt/Hz. We want to achieve an SNR at the demodulator output of at least 50 dB. What is the required transmitter power and channel bandwidth if we employ the following modulation schemes?

1. DSB AM.
2. SSB AM.
3. Conventional AM with a modulation index 0.8

Effect of Noise on Conventional AM (9/12)

- **Example 6.1.2. (Cont'd)** We obtain the PSD of the message process, namely,

$$S_M(f) = \mathscr{F}[R_M(\tau)] = \frac{16}{10{,}000}\Lambda\left(\frac{f}{10{,}000}\right)$$

- $S_M(f)$ is nonzero for $-10{,}000 < f < 10{,}000$; therefore, $W = 10{,}000$ Hz

- We can determine $(S/N)_b$ as a basis of comparison:

$$\left(\frac{S}{N}\right)_b = \frac{P_R}{N_0 W} = \frac{P_R}{2\times10^{-12}\times10^{4}} = \frac{10^8 P_R}{2}$$

- Since the channel attenuation is 50 dB, it follows that

$$P_R = 10^{-5} P_T.$$

Hence,

$$\left(\frac{S}{N}\right)_b = \frac{10^8 P_R}{2} = \frac{10^3 P_T}{2}$$

Effect of Noise on Conventional AM (10/12)

- **Example 6.1.2. (Cont'd)**
- For DSB AM, we have

$$\left(\frac{S}{N}\right)_o = \left(\frac{S}{N}\right)_b = \frac{10^3 P_T}{2} \sim 50\ dB = 10^5.$$

Therefore,

$$P_T = 200\ Watt$$

and

$$\text{BW} = 2W = 20{,}000\ \text{Hz}$$

- For SSB AM,

$$\left(\frac{S}{N}\right)_o = \left(\frac{S}{N}\right)_b = \frac{10^3 P_T}{2} \sim 50\ dB = 10^5 \Rightarrow P_T = 200\ Watt$$

and

$$\text{BW} = W = 10{,}000\ \text{Hz}$$

Effect of Noise on Conventional AM (11/12)

- **Example 6.1.2. (Cont'd)**
- For conventional AM with a=0.8,

$$\left(\frac{S}{N}\right)_o = \eta\left(\frac{S}{N}\right)_b = \eta\frac{10^3 P_T}{2},$$

where η is the modulation efficiency given by

$$\eta = \frac{a^2 P_{M_n}}{1 + a^2 P_{M_n}}.$$

- Since $\max|m(t)|$=6, we have

$$P_{M_n} = \frac{P_M}{(\max|m(t)|)^2} = \frac{P_M}{36}.$$

To determine P_M, we have

$$P_M = R_M(\tau)|_{\tau=0} = 16;$$

therefore,

$$P_{M_n} = \frac{P_M}{(\max|m(t)|)^2} = \frac{16}{36} = \frac{4}{9}$$

Effect of Noise on Conventional AM (12/12)

- **Example 6.1.2. (Cont'd)**
- Hence,

$$\eta = \frac{a^2 P_{M_n}}{1 + a^2 P_{M_n}} \approx 0.22$$

Therefore,

$$\left(\frac{S}{N}\right)_o \approx 0.22 \frac{10^3 P_T}{2} = 0.11 \times 10^3 P_T = 10^5$$

or

$$P_T \approx 909 \ Watt.$$

- The bandwidth of conventional AM is equal to the bandwidth of DSB AM, i.e.,

$$\text{BW} = 2W = 20 \text{ kHz}$$