Chapter 3 Amplitude Modulation (I)

Introduction to Modulation (1/2)

- The analog signal to be transmitted is denoted by $m(t)$, which is assumed to be a lowpass signal of bandwidth W; in other words $M(f) \equiv 0$, for $|f| > W$. The power content of this signal is denoted by

$$P_m = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |m(t)|^2 \, dt$$

- The message signal $m(t)$ is transmitted through the communication channel by impressing it on a *carrier* signal of the form

$$c(t) = A_c \cos(2\pi f_c t + \phi_c),$$

where A_c is the carrier amplitude, f_c is the carrier frequency, and ϕ_c is the carrier phase

Introduction to Modulation (2/2)

- Modulation of the carrier $c(t)$ by the message signal $m(t)$ is performed to achieve one or more of the following objectives:
  - To translate the frequency of the lowpass signal to the passband of the channel so that the spectrum of the transmitted bandpass signal will match the passband characteristics of the channel
  - To simplify the structure of the transmitter by employing higher frequencies. For instance, in the transmission of information using electromagnetic waves, transmission of the signal at low frequencies requires huge antennas
  - To accommodate for the simultaneous transmission of signals from several messages sources
  - To expand the bandwidth of the transmitted signal in order to increase its noise and interference immunity in transmission over a noisy channel

Amplitude Modulation (AM) (1/1)

- There are several different ways of amplitude modulating the carrier signal by $m(t)$; each results in different spectral characteristics for the transmitted signal
- We will investigate the following four methods, which is called (a) double sideband, suppressed-carrier AM, (b) conventional double-sideband AM, (c) single-sideband AM, and (d) vestigial-sideband AM

Double-Sideband Suppressed-Carrier AM (1/2)

- A double-sideband, suppressed-carrier (DSB-SC) AM signal is obtained by multiplying the message signal $m(t)$ with the carrier signal $c(t) = A_c \cos(2\pi f_c t)$
- We have the amplitude-modulated signal

$$\begin{aligned} u(t) &= m(t)c(t) \\ &= A_c m(t) \cos(2\pi f_c t) \end{aligned}$$

Double-Sideband Suppressed-Carrier AM (2/2)

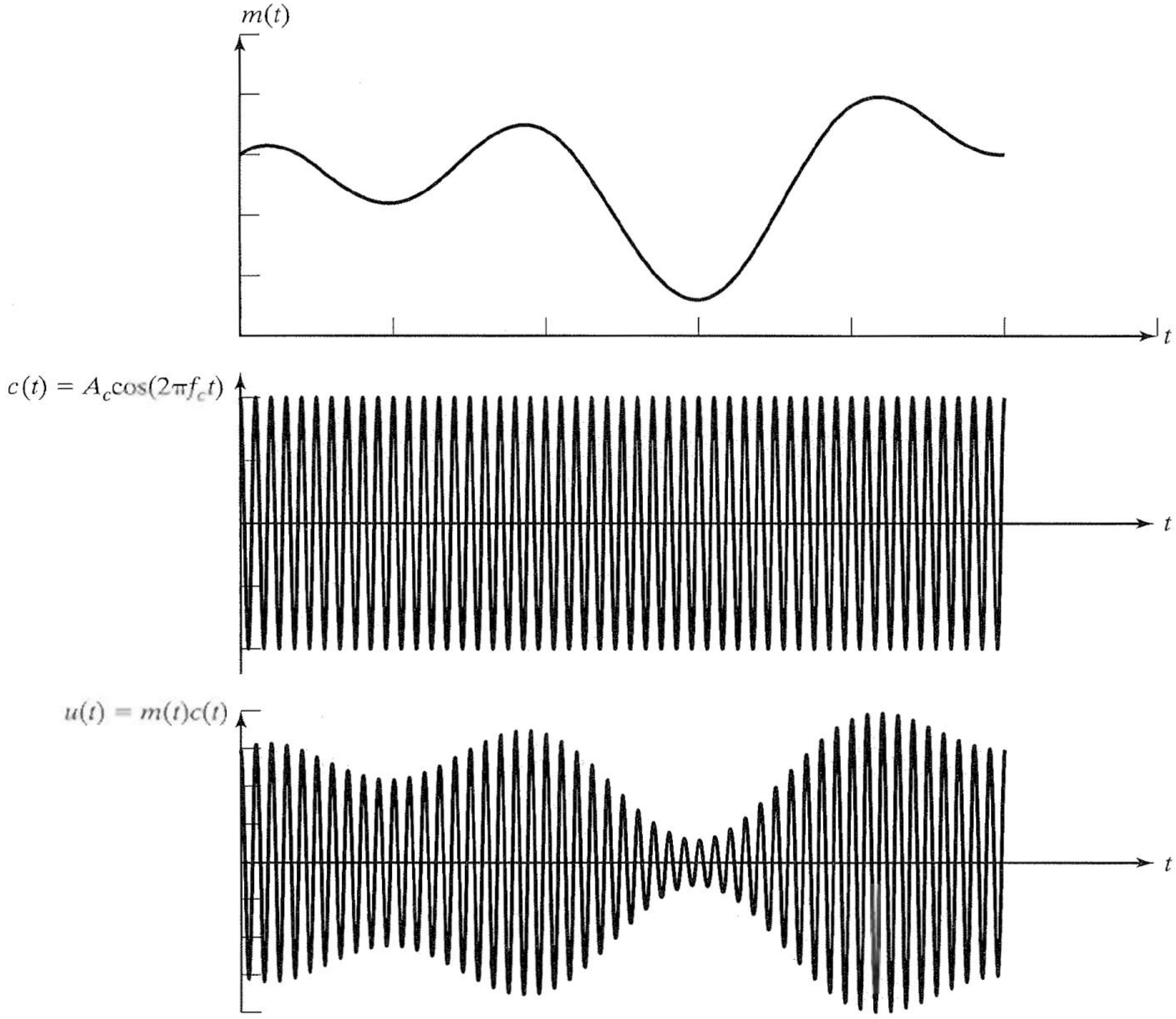

Figure 3.1 An example of message, carrier, and DSB-SC modulated signals.

Spectrum of the DSB-SC AM Signal (1/7)

- The spectrum of the modulated signal can be obtained by taking the Fourier transform of $u(t)$. We obtain

$$U(f) = \frac{A_c}{2}\left[M(f - f_c) + M(f + f_c)\right]$$

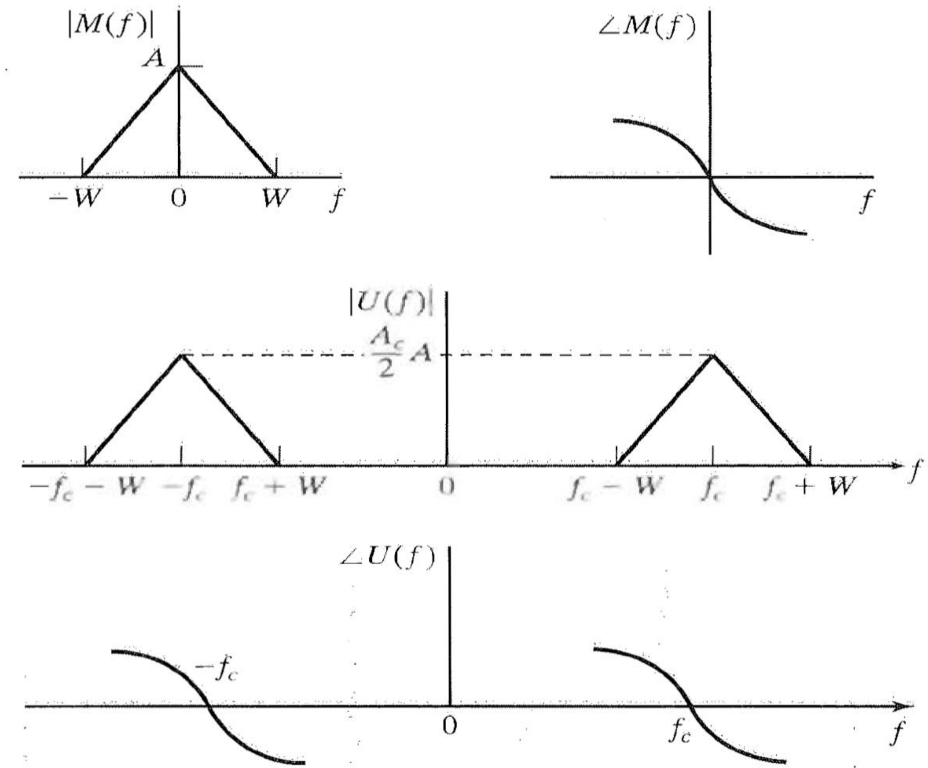

Figure 3.2 Magnitude and phase spectra of the message signal $m(t)$ and the DSB-AM modulated signal $u(t)$.

Spectrum of the DSB-SC AM Signal (2/7)

- The bandwidth occupancy of the amplitude-modulated signal is $2W$, whereas the bandwidth of the message signal $m(t)$ is W
- The channel bandwidth required to transmit the modulated signal $u(t)$ is $B_c = 2W$
- The frequency content of the modulated signal $u(t)$ in the frequency band $|f| > f_c$ is called the upper sideband of $U(f)$, and the frequency content in the frequency band $|f| < f_c$ is called the lower sideband of $U(f)$
- Either one of the sidebands of $U(f)$ contains all the frequency components that are in $M(f)$

Spectrum of the DSB-SC AM Signal (3/7)

- Since $U(f)$ contains both the upper and the lower sidebands, it is called a double-sideband (DSB) AM signal
- The other characteristic of the modulated signal $u(t)$ is that it does not contain a carrier component
- There is no impulse in $U(f)$ at $f=f_c$. For this reason, $u(t)$ is called a suppressed-carrier signal. Therefore, $u(t)$ is a DSB-SC AM signal

Spectrum of the DSB-SC AM Signal (4/7)

- **Example 3.2.1.** Suppose that the modulating signal $m(t)$ is a sinusoid of the form

$$m(t)=a\cos(2\pi f_m t), \quad f_m << f_c.$$

Determine the DSB-SC AM signal and its upper and lower sidebands.

- The DSB-SC AM is expressed in the time domain as

$$u(t) = m(t)c(t) = A_c a\cos(2\pi f_m t)\cos(2\pi f_c t)$$

$$= \frac{A_c a}{2}\cos[2\pi(f_c - f_m)t] + \frac{A_c a}{2}\cos[2\pi(f_c + f_m)t]$$

Spectrum of the DSB-SC AM Signal (5/7)

- **Example 3.2.1.(Cont'd)**
- Taking the Fourier transform, the modulated signal in the frequency domain will have the following form:

$$U(f) = \frac{A_c a}{4}[\delta(f - f_c + f_m) + \delta(f + f_c - f_m)] + \frac{A_c a}{4}[\delta(f - f_c - f_m) + \delta(f + f_c + f_m)].$$

- The lower sideband of $u(t)$ is

$$u_l(t) = \frac{A_c a}{2}\cos[2\pi(f_c - f_m)t].$$

The upper sideband of $u(t)$ is

$$u_u(t) = \frac{A_c a}{2}\cos[2\pi(f_c + f_m)t].$$

Spectrum of the DSB-SC AM Signal (6/7)

- **Example 3.2.1.(Cont'd)**

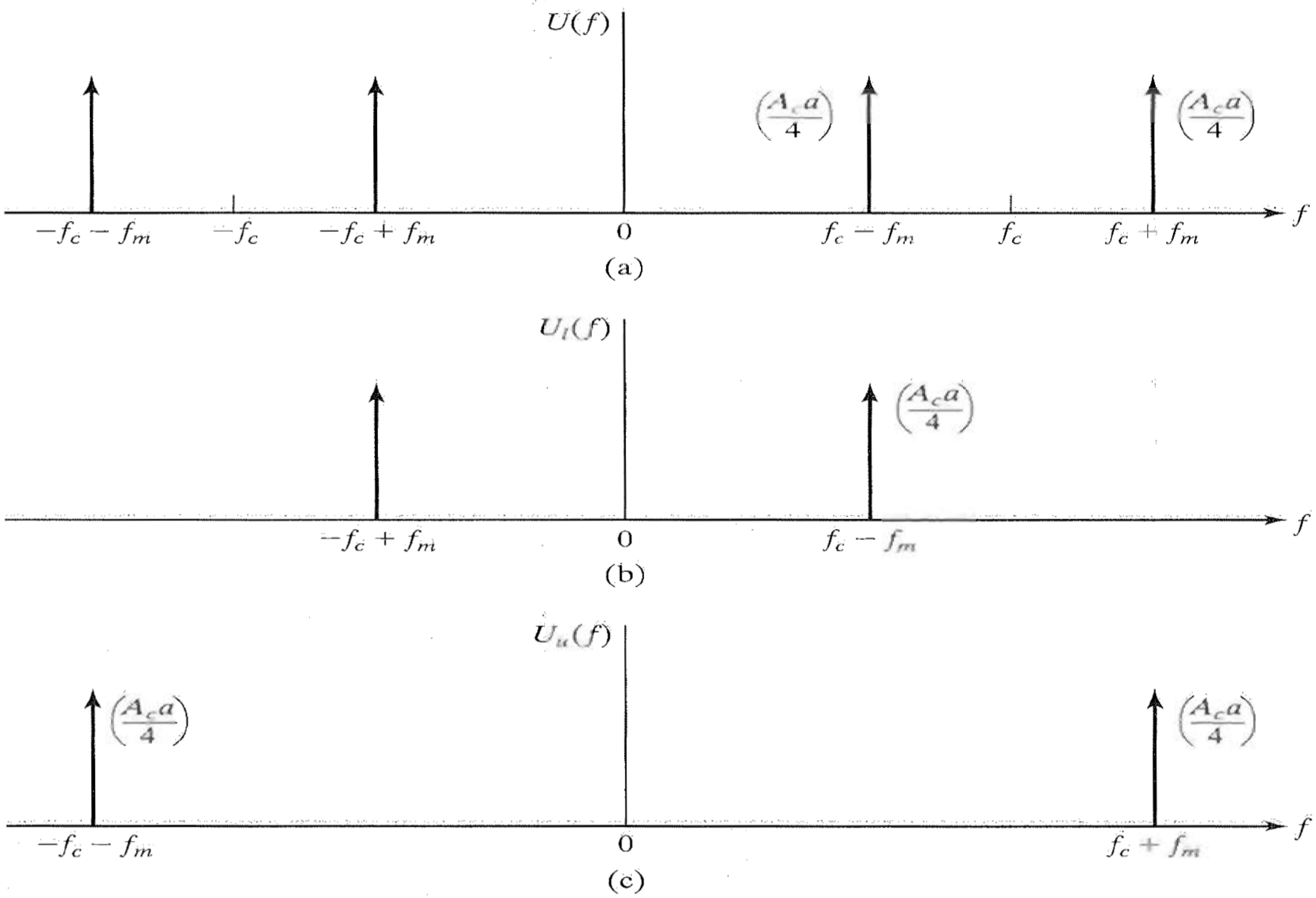

Figure 3.3 (a) The (magnitude) spectrum of a DSB-SC AM signal for a sinusoidal message signal and (b) its lower and (c) upper sidebands.

Spectrum of the DSB-SC AM Signal (7/7)

- **Example 3.2.2.** Let the message signal be $m(t)=\text{sinc}(10^4 t)$. Determine the DSB-SC modulated signal and its bandwidth when the carrier is a sinusoid with a frequency of 1 MHz.
- In this example, $c(t)=\cos(2\pi\times 10^6 t)$. Therefore, $u(t)=\text{sinc}(10^4 t)\cos(2\pi\times 10^6 t)$.
- We have $M(f)=10^{-4}\Pi(10^{-4}f)$. The bandwidth of the message signal is 5000 Hz, and the bandwidth of the modulated signal is twice the bandwidth of the message signal, i.e., 10000 Hz

Power Content of DSB-SC Signals (1/2)

- The power content of the modulated signal is

$$
\begin{aligned}
P_u &= \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} u^2(t)\,dt \\
&= \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} A_c^2 m^2(t)\cos^2(2\pi f_c t)\,dt \\
&= \frac{A_c^2}{2}\lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} m^2(t)[1+\cos(4\pi f_c t)]\,dt \quad (3.2.1) \\
&= \frac{A_c^2}{2} P_m
\end{aligned}
$$

- P_m indicates the power in the message signal $m(t)$

Power Content of DSB-SC Signals (2/2)

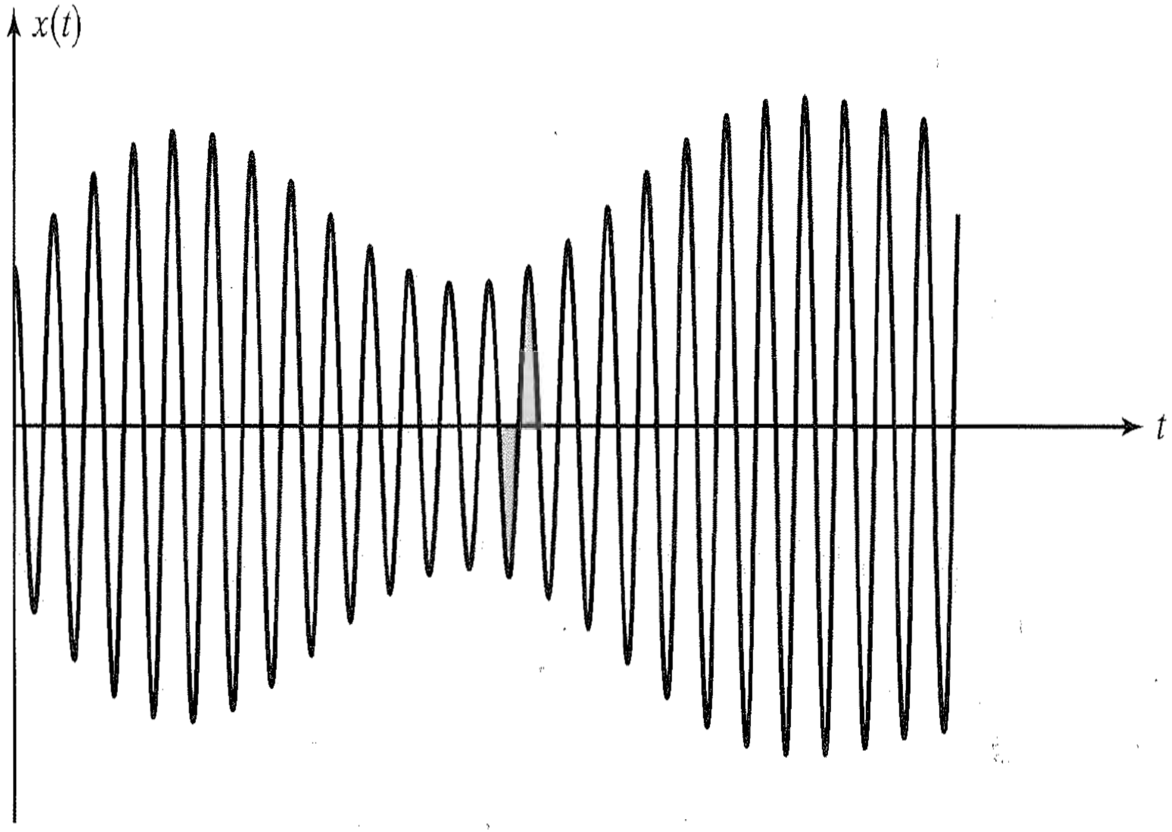

Figure 3.6 This figure shows why the second term in Equation (3.2.1) is zero.

Demodulation of DSB-SC AM Signals (1/6)

- Suppose that the DSB-SC AM signal $u(t)$ is transmitted through an ideal channel (with no channel distortion and no noise). Then the received signal is equal to the modulated signal, i.e.,

$$\begin{aligned} r(t) &= u(t) \\ &= A_c m(t)\cos(2\pi f_c t) \end{aligned}$$

- The received signal is first multiplied by a locally generated sinusoid $\cos(2\pi f_c t+\phi)$, where ϕ is the phase of the sinusoid. The multiplication of $r(t)$ with $\cos(2\pi f_c t+\phi)$ yields

$$\begin{aligned} r(t)\cos(2\pi f_c t+\phi) &= A_c m(t)\cos(2\pi f_c t)\cos(2\pi f_c t+\phi) \\ &= \frac{1}{2}A_c m(t)\cos(\phi)+\frac{1}{2}A_c m(t)\cos(4\pi f_c t+\phi) \end{aligned}$$

Demodulation of DSB-SC AM Signals (2/6)

- The frequency content of the message signal $m(t)$ is limited to W , where $W \ll f_c$, the lowpass filter can be designed to eliminate the signal component centered at frequency $2f_c$ and to pass the signal components centered at frequency $f=0$ without experiencing distortion
- The output of the ideal lowpass filter is

$$y_l(t) = \frac{1}{2} A_c m(t) \cos(\phi)$$

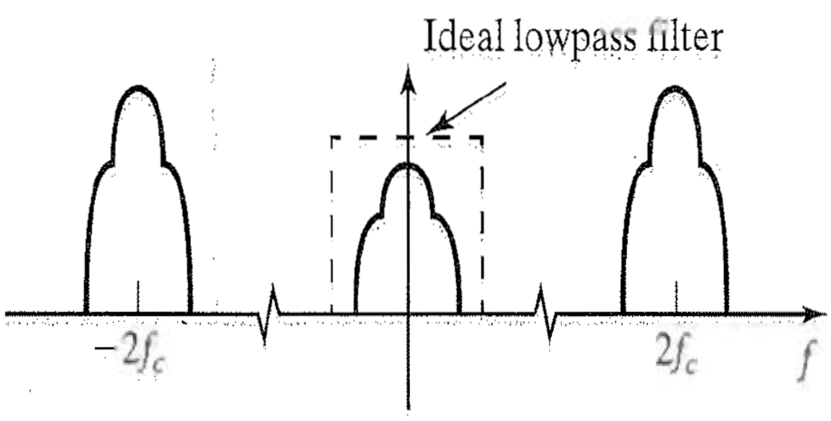

Figure 3.7 Frequency-domain representation of the DSB-SC AM demodulation.

Demodulation of DSB-SC AM Signals (3/6)

- Note that $m(t)$ is multiplied by $\cos(\phi)$; therefore, the power in the demodulated signal is decreased by a factor of $\cos^2(\phi)$
- When $\phi \neq 0$, the amplitude of the desired signal is reduced by the factor $\cos(\phi)$. If $\phi = 45^o$, the amplitude of the desired signal is reduced by $\sqrt{2}$ and the signal power is reduced by a factor of two. If $\phi = 90^o$, the desired signal component vanishes
- The preceding discussion demonstrates the need for a *phase-coherent* or *synchronous demodulator*, the phase of the locally generated sinusoid is identical to that of the incoming signal, for recovering the message signal $m(t)$ from the received signal.

Demodulation of DSB-SC AM Signals (4/6)

- One method to add a carrier component into the transmitted signal is shown in Fig. 3.8. We call such a carrier component "a pilot tone".
- The amplitude of the pilot tone A_p and its power $A_p^2/2$ are selected to be significantly smaller than those of the modulated signal $u(t)$

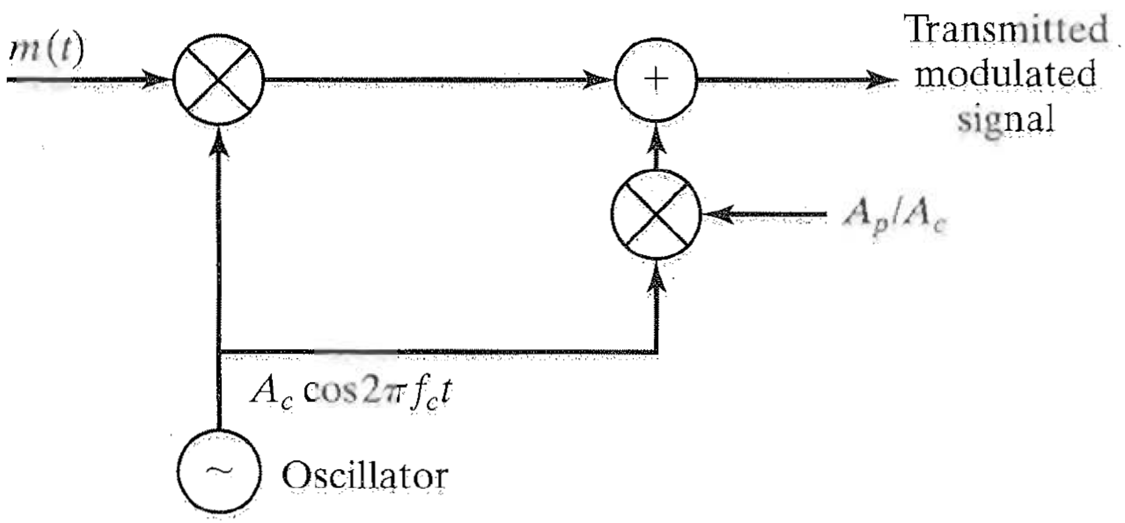

Figure 3.8 Addition of a pilot tone to a DSB-AM signal.

Demodulation of DSB-SC AM Signals (5/6)

- The transmitted signal is double-sideband, but it is no longer a suppressed carrier signal
- At the receiver, a narrow band filter tuned to frequency f_c filters out the pilot signal component; its output is used to multiply the received signal

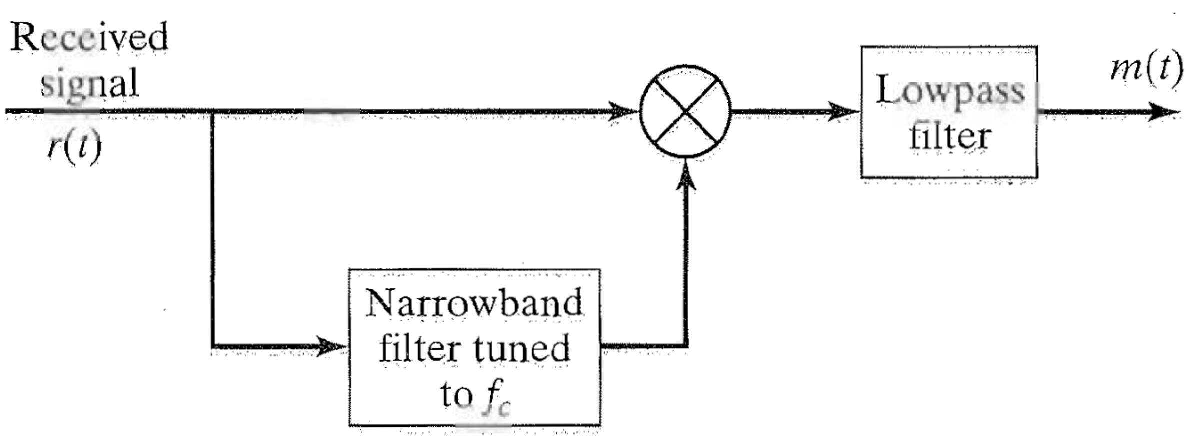

Figure 3.9 Use of a pilot tone to demodulate a DSB-AM signal.

Demodulation of DSB-SC AM Signals (6/6)

- The presence of the pilot signal results in a DC component in the demodulated signal; this must be subtracted out in order to recover $m(t)$
- Adding a pilot tone to the transmitted signal requires that a certain portion of the transmitted signal power be allocated to the transmission of the pilot
- We can use a phase-locked loop to generate a phase-locked sinusoidal carrier from the received signal $r(t)$

Conventional Amplitude Modulation (1/3)

- The transmitted signal of a conventional AM signal is expressed as

$$u(t) = A_c[1 + m(t)]\cos(2\pi f_c t),$$

where $|m(t)| \leq 1$

- If $m(t)$<-1 for some t, the AM signal is *overmodulated* and its demodulation is rendered more complex. In practice, $m(t)$ is scaled so that its magnitude is always less than unity

Conventional Amplitude Modulation (2/3)

- The existence of this extra carrier results in a very simple structure for the demodulator. That is why commercial AM broadcasting generally employs this type of modulation

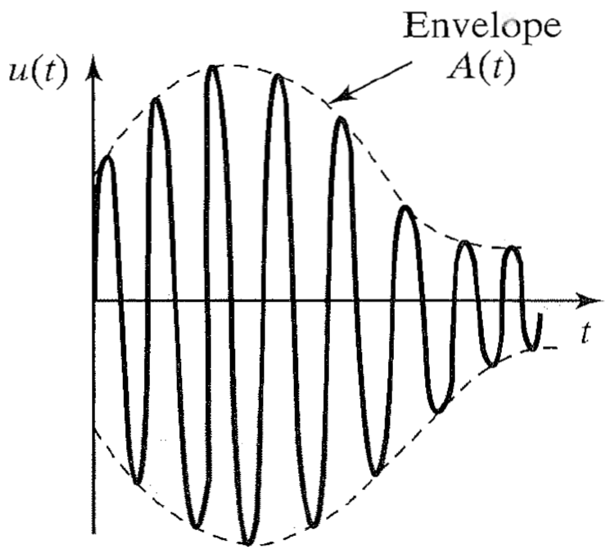

Figure 3.10 A conventional AM signal in the time domain.

Conventional Amplitude Modulation (3/3)

- It is sometimes convenient to express $m(t)$ as

$$m(t) = a m_n(t)$$

where $m_n(t)$ is normalized such that its minimum value is -1. This can be done by defining

$$m_n(t) = \frac{m(t)}{\max |m(t)|}$$

- a is called the modulation index, which is generally a constant less than 1
- The modulated signal can also be represented as

$$u(t) = A_c[1 + a m_n(t)]\cos(2\pi f_c t)$$

Spectrum of the Conventional AM Signal (1/5)

- If $m(t)$ is a message signal with Fourier transform (spectrum) $M(f)$, the spectrum of the amplitude-modulated signal $u(t)$ is

$$U(f) = \mathscr{F}[A_c a m_n(t)\cos(2\pi f_c t)] + \mathscr{F}[A_c \cos(2\pi f_c t)]$$

$$= \frac{A_c a}{2}[M_n(f - f_c) + M_n(f + f_c)] + \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)]$$

- The spectrum of a conventional AM signal occupies a bandwidth twice the bandwidth of the message signal

Spectrum of the Conventional AM Signal (2/5)

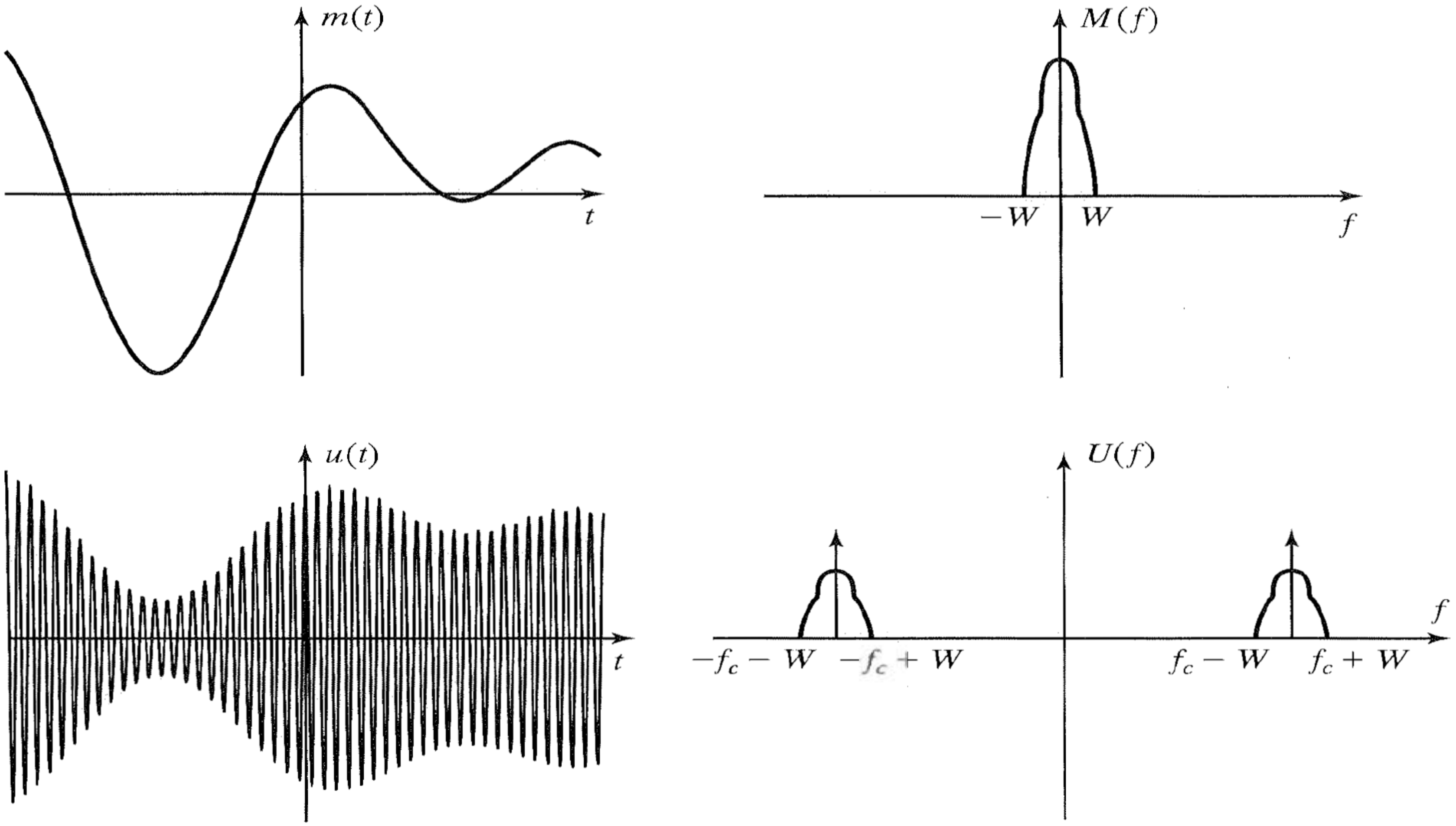

Figure 3.11 Conventional AM in both the time and frequency domain.

Spectrum of the Conventional AM Signal (3/5)

- **Example 3.2.4.** Suppose that the normalized modulating signal $m_n(t)$ is a sinusoid of the form

$$m_n(t)=\cos(2\pi f_m t), f_m << f_c.$$

Determine the conventional AM signal, its upper and lower sidebands, and its spectrum, assuming a modulation index a

- The conventional AM signal is expressed as

$$\begin{aligned} u(t) &= A_c[1+a\cos(2\pi f_m t)]\cos(2\pi f_c t) \\ &= A_c\cos(2\pi f_c t) + \frac{A_c a}{2}\cos[2\pi(f_c - f_m)t] \\ &\quad + \frac{A_c a}{2}\cos[2\pi(f_c + f_m)t] \end{aligned}$$

Spectrum of the Conventional AM Signal (4/5)

- **Example 3.2.4. (Cont'd)** The lower sideband component is

$$u_l(t) = \frac{A_c a}{2}\cos[2\pi(f_c - f_m)t]$$

while the upper sideband component is

$$u_u(t) = \frac{A_c a}{2}\cos[2\pi(f_c + f_m)t]$$

The spectrum of the conventional AM signal $u(t)$ is

$$\begin{aligned} U(f) = {} & \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] \\ & + \frac{A_c a}{4}\left[\delta(f - f_c + f_m) + \delta(f + f_c - f_m)\right] \\ & + \frac{A_c a}{4}\left[\delta(f - f_c - f_m) + \delta(f + f_c + f_m)\right] \end{aligned}$$

Spectrum of the Conventional AM Signal (5/5)

- **Example 3.2.4. (Cont'd)**
- It is interesting to note that the power of the carrier component which is $A_c^2/2$, exceeds the total power $(A_c^2 a^2/4)$ of the two sidebands because $a<1$

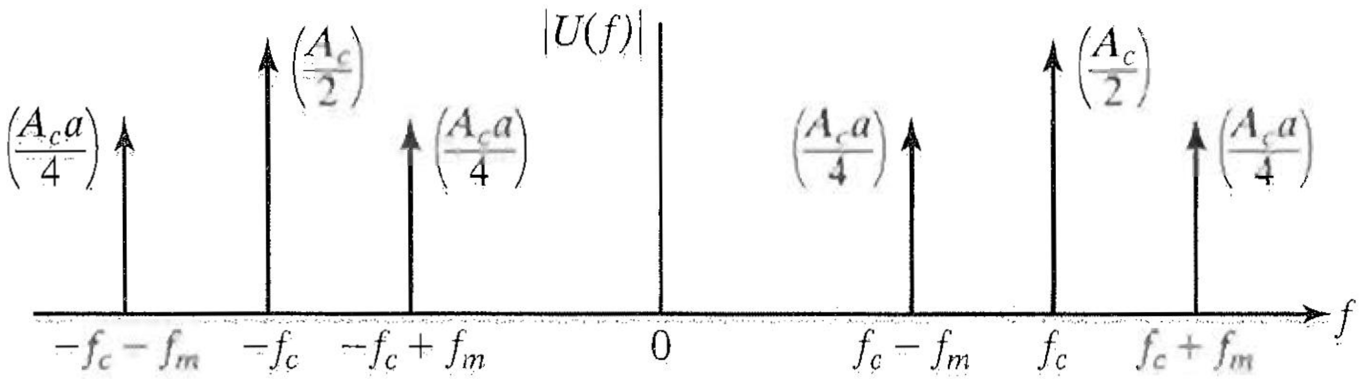

Figure 3.12 Spectrum of a DSB-AM signal in Example 3.2.4.

Power for the Conventional AM Signal (1/5)

- The power of a conventional AM-modulated signal is

$$P_u = \frac{A_c^2}{2} + \frac{A_c^2}{2} a^2 P_{m_n},$$

where P_{m_n} is the power of $m_n(t)$

- The first component in the preceding relation applies to the existence of the carrier, and this component does not carry any information
- The second component is the information-carrying component which is usually much smaller than the first component ($a<1$, $|m_n(t)|<1$, and for signals with a large dynamic rang, $P_{m_n} << 1$)

Power for the Conventional AM Signal (2/5)

- The conventional AM systems are far less power efficient than the DSB-SC systems
- The advantage of conventional AM is that it can be easily demodulated

Power for the Conventional AM Signal (3/5)

- **Example 3.2.5.** The signal $m(t) = 3\cos(200\pi t) + \sin(600\pi t)$ is used to modulate the carrier $c(t) = \cos(2\times10^5 t)$. The modulation index is $a = 0.85$. Determine the power in the carrier component and in the sideband components of the modulated signal

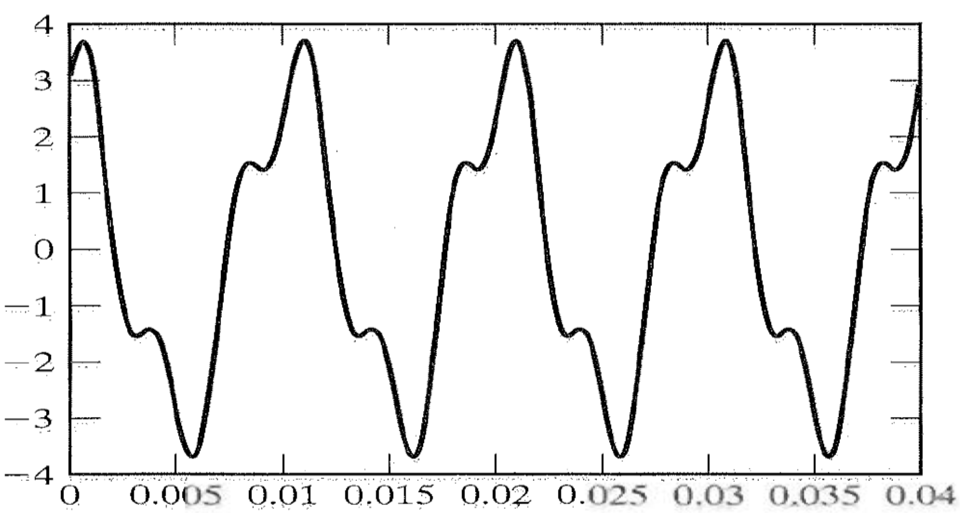

Figure 3.13 The message signal in Example 3.2.5.

Power for the Conventional AM Signal (4/5)

- **Example 3.2.5. (Cont'd)** We first determine the normalized signal, $m_n(t)$. In order to find $m_n(t)$, we have to determine $\max|m(t)|$. We find its maximum is 3.6955. Therefore,

$$m_n(t) = \frac{3\cos(200\pi t) + \sin(600\pi t)}{3.6955}$$
$$= 0.8118\cos(200\pi t) + 0.2706\sin(600\pi t)$$

- The power in $m_n(t)$ is

$$P_{m_n} = \tfrac{1}{2}[0.8118^2 + 0.2706^2]$$
$$= 0.3661$$

- The power in the carrier component of the modulated signal is $$\frac{A_c^2}{2} = 0.5$$

Power for the Conventional AM Signal (5/5)

- **Example 3.2.5. (Cont'd)** The power in the sideband is

$$\frac{A_c^2}{2} a^2 P_{m_n} = \frac{1}{2} \times 0.85^2 \times 0.3661$$
$$= 0.1323$$

Demodulation of Conventional DSB-AM (Conventional AM) Signals (1/3)

- The major advantage of conventional AM signal transmission is the ease of demodulation
- There is no need for a synchronous demodulator
- Since the message signal $m(t)$ satisfies the condition $|m_n(t)|<1$, the envelop (amplitude) $1+m_n(t)>0$
- If we rectify the received signal, we eliminate the negative values without affecting the message signal
- The message signal is recovered by passing the received signal through a lowpass filter whose bandwidth matches that of the message signal

Demodulation of Conventional DSB-AM (Conventional AM) Signals (2/3)

- The combination of the rectifier and the lowpass filter is called an *envelope detector*
- The output of the envelope detector is of the form

$$z(t)=a_c+g_2m(t)$$

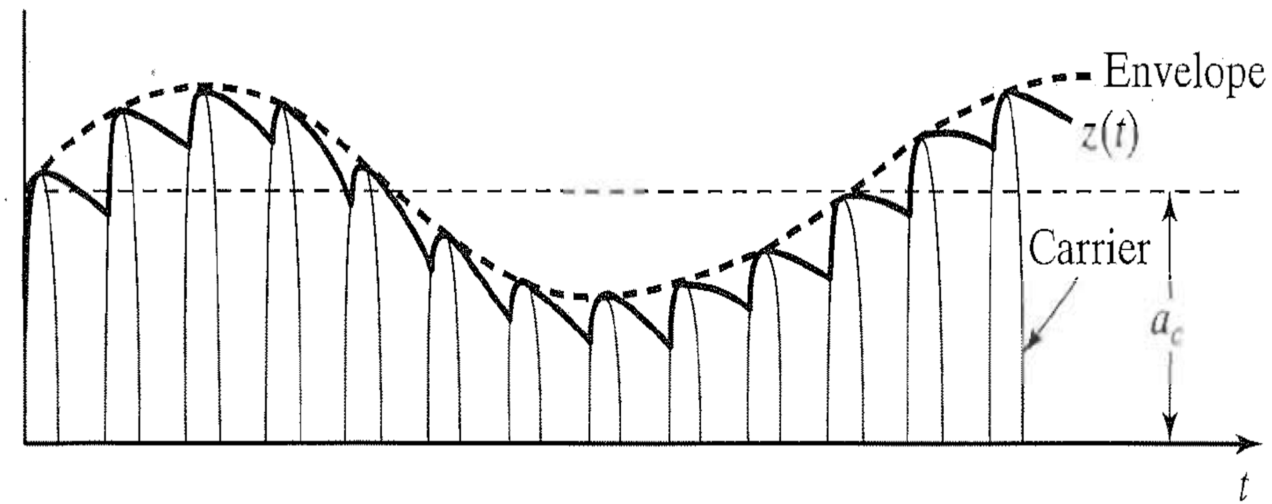

Figure 3.14 Envelope detection of a conventional AM signal.

Demodulation of Conventional DSB-AM (Conventional AM) Signals (3/3)

- The simplicity of the demodulator has made conventional DSB-AM a practical choice for AM-radio broadcasting
- The power inefficiency of conventional AM is justified by the fact that there are few broadcast transmitters relative to the number of receivers
- It is cost-effective to construct powerful transmitters and sacrifice power efficiency in order to simplify the signal demodulation at the receivers

Single-Sideband AM (1/4)

- The two sidebands of a DSB-SC AM signal are redundant. One sideband is enough for transmitting the baseband message $m(t)$
- A single-sideband (SSB) AM signal is represented mathematically as

$$u(t) = A_c m(t)\cos(2\pi f_c t) \mp A_c \hat{m}(t)\sin(2\pi f_c t)$$

where $\hat{m}(t)$ is the Hilbert transform of $m(t)$, the plus and minus sign determines which sideband we obtain

- The plus sign indicates the lower sideband, and the minus sign indicates the upper sideband

Single-Sideband AM (2/4)

- The Hilbert transform may be viewed as a linear filter with impulse response $h(t)=1/\pi t$ and frequency response

$$H(f)=\begin{cases} -j, & f>0 \\ j, & f<0 \\ 0, & f=0 \end{cases}$$

Single-Sideband AM (3/4)

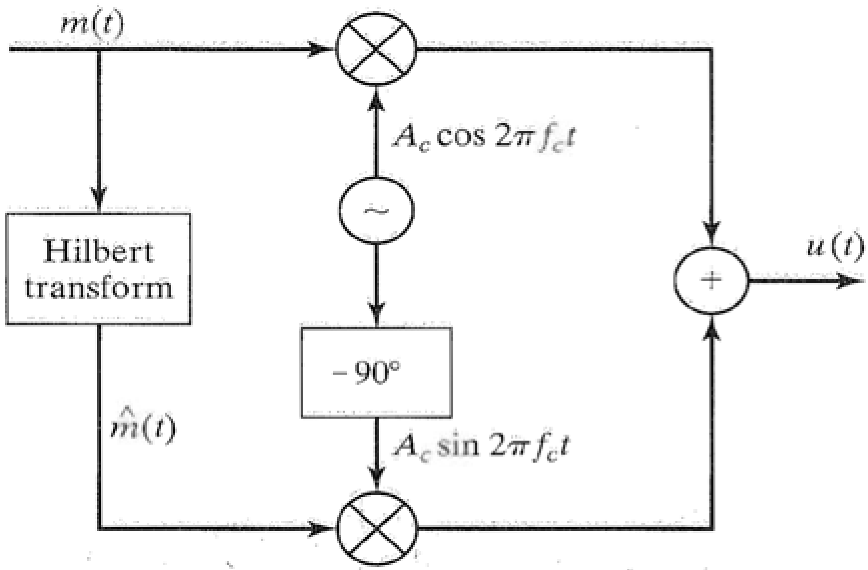

Figure 3.15 Generation of a lower single-sideband AM signal.

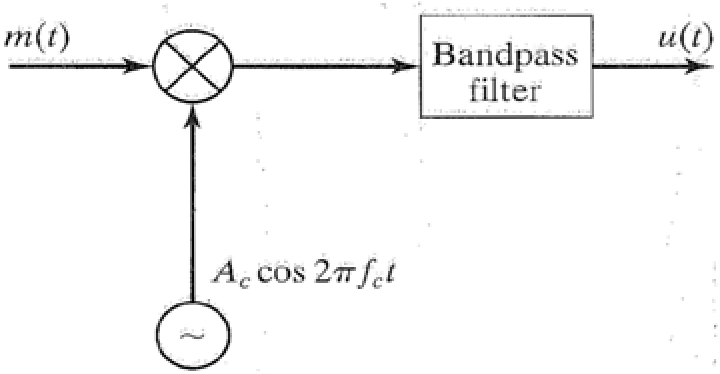

Figure 3.16 Generation of a single-sideband AM signal by filtering one of the sidebands of a DSB-SC AM signal.

Single-Sideband AM (4/4)

- **Example 3.2.6.** Suppose that the modulating signal is a sinusoid of the form

$$m(t) = \cos(2\pi f_m t), \quad f_m \ll f_c.$$

Determine the two possible SSB-AM signals

- The Hilbert transform of $m(t)$ is

$$\hat{m}(t) = \sin(2\pi f_m t).$$

Hence, $u(t) = A_c \cos(2\pi f_m t)\cos(2\pi f_c t) \mp A_c \sin(2\pi f_m t)\sin(2\pi f_c t)$.

We obtain the upper-sideband signal

$$u_u(t) = A_c \cos(2\pi (f_c + f_m)t),$$

and the lower-sideband signal

$$u_l(t) = A_c \cos(2\pi (f_c - f_m)t).$$

Derivation of the Expression for SSB-AM Signals (1/4)

- Let $m(t)$ be a signal with the Fourier transform (spectrum) $M(f)$. An upper single-sideband amplitude-modulated signal (USSB AM) is obtained by eliminating the lower sideband of a DSB amplitude-modulated signal $u_{DSB}(t)=2A_c m(t)\cos(2\pi f_c t)$
- We pass $u_{DSB}(t)$ through a highpass filter whose transfer function is given by

$$H(f)=\begin{cases}1, & |f|>f_c\\ 0, & otherwise\end{cases}.$$

$H(f)$ can be written as

$$H(f)=u_{-1}(f-f_c)+u_{-1}(-f-f_c).$$

$u_{-1}(.)$ represents the unit-step function

(Note: $u_{-1}(-f-f_c)=1 \Rightarrow -f-f_c>0 \Rightarrow f<-f_c$)

Derivation of the Expression for SSB-AM Signals (2/4)

- The spectrum of the USSB-AM signal is given by

$$U_u(f) = A_c M(f - f_c) u_{-1}(f - f_c) + A_c M(f + f_c) u_{-1}(-f - f_c)$$

or equivalently,

$$U_u(f) = A_c M(f) u_{-1}(f)\big|_{f=f-f_c} + A_c M(f) u_{-1}(-f)\big|_{f=f+f_c} \quad (3A.1)$$

- Taking the inverse Fourier transform of both sides of (3A.1), we obtain

$$u_u(t) = [A_c m(t) \bigstar \mathscr{F}^{-1}[u_{-1}(f)]] e^{j2\pi f_c t} + [A_c m(t) \bigstar \mathscr{F}^{-1}[u_{-1}(-f)]] e^{-j2\pi f_c t} \quad (3A.2)$$

- Note that

$$\mathscr{F}\left[\frac{1}{2}\delta(t) + \frac{j}{2\pi t}\right] = u_{-1}(f)$$

$$\mathscr{F}\left[\frac{1}{2}\delta(t) - \frac{j}{2\pi t}\right] = u_{-1}(-f) \quad (3A.3)$$

Derivation of the Expression for SSB-AM Signals (3/4)

- Substituting (3A.3) in (3A.2), we obtain

$$u_u(t) = A_c m(t) \star \left[\tfrac{1}{2}\delta(t) + \tfrac{j}{2\pi t}\right] e^{j2\pi f_c t} + A_c m(t) \star \left[\tfrac{1}{2}\delta(t) - \tfrac{j}{2\pi t}\right] e^{-j2\pi f_c t}$$

$$= \frac{A_c}{2}\left[m(t) + j\hat{m}(t)\right] e^{j2\pi f_c t} + \frac{A_c}{2}\left[m(t) - j\hat{m}(t)\right] e^{-j2\pi f_c t} \quad (3A.4)$$

- Using Euler's relations in (3A.4), we obtain

$$u_u(t) = A_c m(t)\cos(2\pi f_c t) - A_c \hat{m}(t)\sin(2\pi f_c t)$$

Derivation of the Expression for SSB-AM Signals (4/4)

- The expression for the LSSB-AM signal can be derived by noting that

$$u_u(t) + u_l(t) = u_{DSB}(t)$$

Therefore,

$$u_l(t) = A_c m(t)\cos(2\pi f_c t) + A_c \hat{m}(t)\sin(2\pi f_c t)$$

Thus, the time-domain representation of a SSB-AM signal can generally be expressed as

$$u_{SSB}(t) = A_c m(t)\cos(2\pi f_c t) \mp A_c \hat{m}(t)\sin(2\pi f_c t), \quad (3A.7)$$

where the minus sign corresponds to the USSB-AM signal, and the plus sign corresponds to the LSSB-AM signal

Demodulation of SSB-AM Signals (1/2)

- To recover the message signal $m(t)$ in the received SSB-AM signal, we require a phase-coherent or synchronous demodulator, as was the case for DSB-SC AM signals
- For the USSB signal given in (3A.7), we have

$$r(t)\cos(2\pi f_c t+\phi)=u(t)\cos(2\pi f_c t+\phi)$$
$$=\frac{1}{2}A_c m(t)\cos(\phi)+\frac{1}{2}A_c \hat{m}(t)\sin(\phi)$$
$$+\text{double frequency terms.}$$

By passing the product signal through an ideal LPF, the double-frequency components are eliminates, leaving us with

$$y_l(t)=\frac{1}{2}A_c m(t)\cos(\phi)+\frac{1}{2}A_c \hat{m}(t)\sin(\phi)$$

- If $\phi=0$, $y_l(t)$ becomes $\frac{1}{2}A_c m(t)$

Demodulation of SSB-AM Signals (2/2)

- The transmission of a pilot tone at the carrier frequency is a very effective method for providing a phase-coherent reference signal for performing synchronous demodulation at the receiver
- The spectral efficiency of SSB-AM makes this modulation very attractive for use in voice communications over telephone channels
- When $m(t)$ has a large power concentrated in the vicinity of $f=0$, the sideband filter must have an extremely sharp cutoff in the vicinity of the carrier in order to reject the second sideband. Such filter characteristics are very difficult to implement in practice

Vestigial-Sideband AM (1/7)

- A vestigial-sideband system can be relaxed by allowing vestige, which is a portion of the unwanted sideband, to appear at the output of the modulator
- VSB AM is used in standard analog TV broadcasting
- In the time domain, the VSB signal may be expressed as

$$u(t) = [A_c m(t)\cos(2\pi f_c t)] \star h(t)$$

where $h(t)$ is the impulse response of the VSB filter. In the frequency domain, the corresponding expression is

$$U(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]H(f) \quad (3.2.15)$$

Vestigial-Sideband AM (2/7)

- To demodulate the VSB signal $u(t)$, we multiply $u(t)$ by the carrier component $\cos(2\pi f_c t)$ and pass the result through an ideal lowpass filter
- The product signal is

$$v(t) = u(t)\cos(2\pi f_c t),$$

or equivalently,

$$V(f) = \frac{1}{2}\left[U(f - f_c) + U(f + f_c)\right] \quad (3.2.16)$$

- Substitute (3.2.15) into (3.2.16), we obtain

$$V(f) = \frac{A_c}{4}\left[M(f - 2f_c) + M(f)\right]H(f - f_c) + \frac{A_c}{4}\left[M(f) + M(f + 2f_c)\right]H(f + f_c)$$

Vestigial-Sideband AM (3/7)

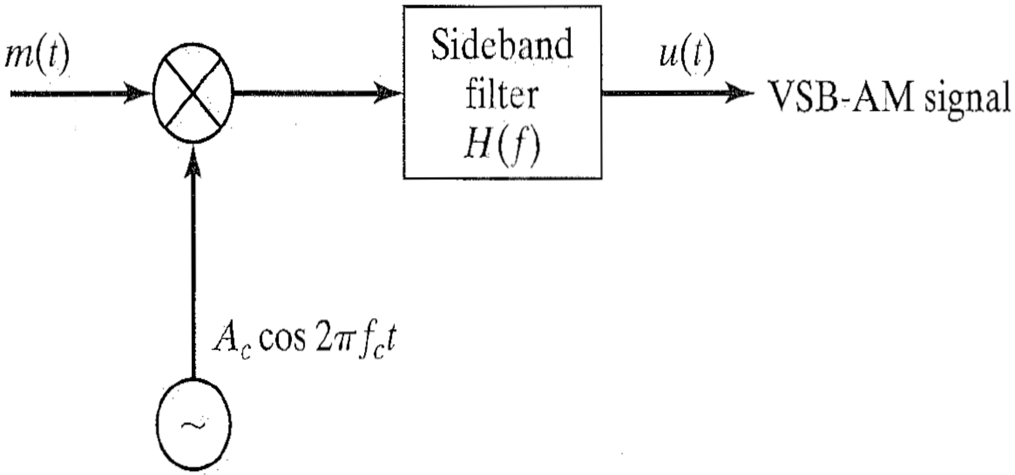

Figure 3.17 Generation of vestigial-sideband AM signal.

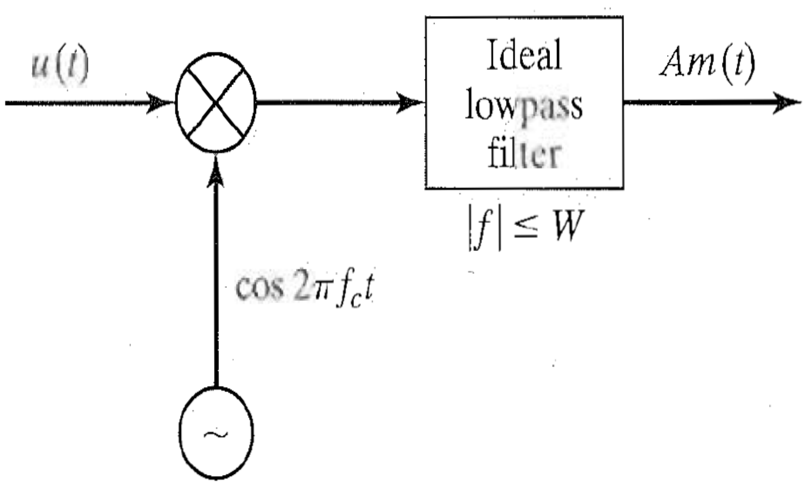

Figure 3.18 Demodulation of VSB signal.

Vestigial-Sideband AM (4/7)

- The lowpass filter rejects the double-frequency terms and passes only the components in the frequency range $|f| \leqq W$
- The signal spectrum at the output of the ideal lowpass filter is

$$V_l(f) = \frac{A_c}{4} M(f)\left[H(f - f_c) + H(f + f_c)\right]$$

- The message signal at the output of the lowpass filter must be undistorted. Hence, the VSB-filter characteristic must satisfy the condition

$$H(f - f_c) + H(f + f_c) = \text{constant}, |f| \leq W$$

- $H(f)$ in Fig. 3.19 selects the upper sideband and a vestige of the lower sideband of $U(f)$

Vestigial-Sideband AM (5/7)

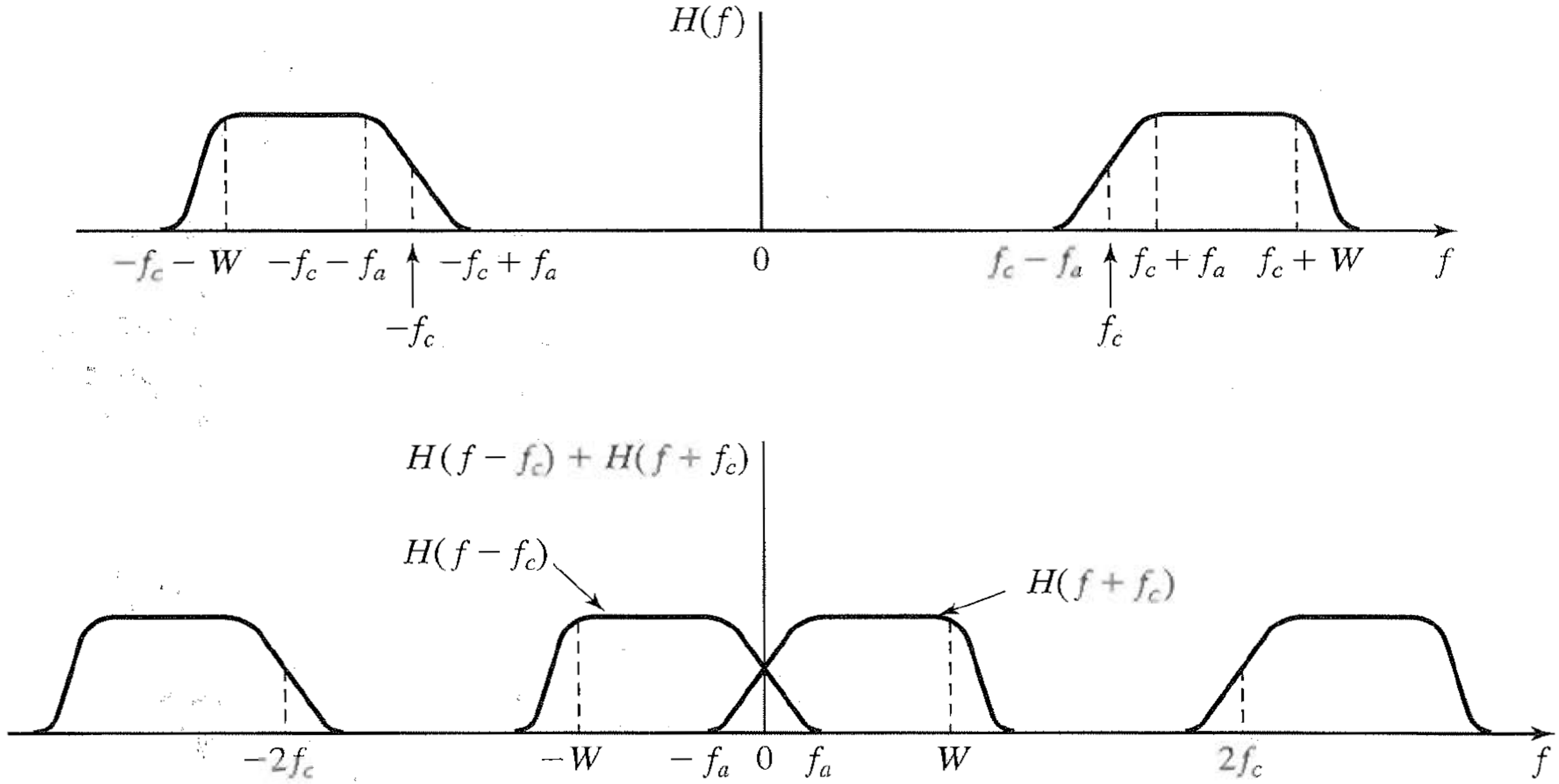

Figure 3.19 VSB-filter characteristics.

Vestigial-Sideband AM (6/7)

- To avoid distortion of the message signal, the VSB filter in Fig. 3.19 should have a linear phase over its passband

$$f_c - f_a \leqq |f| \leqq f_c + W$$

- $H(f)$ in Fig. 3.20 selects the lower sideband and a vestige of the upper sideband of $U(f)$

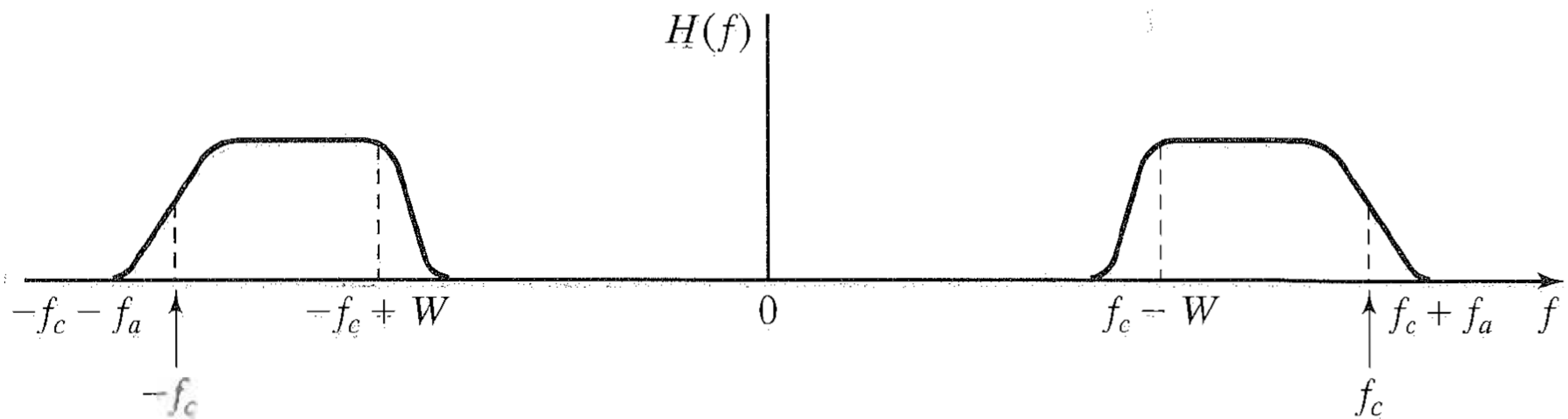

Figure 3.20 Frequency response of the VSB filter for selecting the lower sideband of the message signals.

Vestigial-Sideband AM (7/7)

- **Example 3.2.7.** Suppose that the message signal is given as

$$m(t) = 10 + 4\cos(2\pi t) + 8\cos(4\pi t) + 10\cos(20\pi t).$$

  Specify the frequency response characteristic of a VSB filter

- The VSB filter can be designed as follows

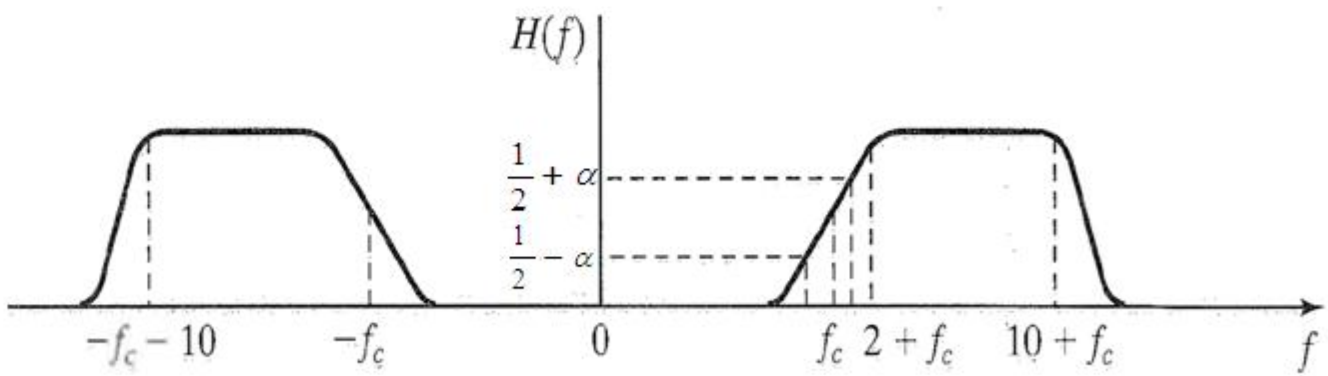

Figure 3.21 Frequency-response characteristics of the VSB filter in Example 3.2.7.