





Electron (電子): a particle of a negative charge $(-1.6 \times 10^{-19} \text{ C})$. Hole (電洞): a fictitious (虛構的) particle of a positive charge $(+1.6 \times 10^{-19} \text{ C})$.









 E_C

13, jswu

 E_c

 E_{a}

E. • • • •

T = 0 K

T = 0 K

n-type

p-type

- Acceptor (or, Acceptor-like) level:
 if filled by an electron _____
 - if empty
- → negatively charged (帶一電)
 → electrically neutral (電中性)
- ◆ Activation energy (活化能量), or lonization energy (雞子化能量)



Position of acceptor atom

When a large number of dopant atoms are present





14, jswu

Carrier Concentration at Thermal Equilibrium (熱平衡)

Ε

- ◆ Carrier concentration (載子濃度): Electron concentration (電子濃度), *n* Hole concentration (電洞濃度), *p*
- For a practical semiconductor:

$$n = \mathbf{N}_{\mathrm{C}} \cdot F_{\frac{1}{2}}\left(\frac{E_{\mathrm{F}} - E_{\mathrm{C}}}{kT}\right) \quad \& \quad p = \mathbf{N}_{\mathrm{V}} \cdot F_{\frac{1}{2}}\left(-\frac{E_{\mathrm{F}} - E_{\mathrm{V}}}{kT}\right)$$

where N_C is effective density of states in the conduction band (導電帶有效能態密度),

- N_V is effective density of states in the valence band (價電帶有效能態密度),
- $F_{\frac{1}{2}}(\mathbf{x})$ is Fermi integral (費米積分) with $n = \frac{1}{2}$,

Fermi integral:
$$F_n(x) = \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{y^n}{1 + \exp(y - x)} \, dy$$
 (with $n > -1$)
In the case of calculating carrier concentration, we have $n = \frac{1}{2}$.

- E_F is Fermi level (費米準位),
- k is Boltzmann's constant (波茲曼常數) (= 8.62×10⁻⁵ eV/K = 1.38×10⁻²³ J/K), T is absolute temperature (絕對溫度).

 $n = N_{\rm C} \cdot F_{\frac{1}{2}}(\frac{E_F - E_C}{kT}) \quad \text{where } N_{\rm C} \text{ is effective density of states in the conduction band.}$ $p = N_{\rm V} \cdot F_{\frac{1}{2}}(-\frac{E_F - E_V}{kT}) \quad \text{where } N_{\rm V} \text{ is effective density of states in the valence band.}$

◆ If *E*_C-3kT > *E*_F > *E*_V+3kT, semiconductor is called *nondegenerate* (非簡併的).

$$n = N_{\rm C} F_{1/2} \left(\frac{E_{\rm F} - E_{\rm C}}{kT}\right) \approx N_{\rm C} \exp\left(\frac{E_{\rm F} - E_{\rm C}}{kT}\right)$$

$$p = N_{\rm V} F_{1/2} \left(-\frac{E_{\rm F} - E_{\rm V}}{kT}\right) \approx N_{\rm V} \exp\left(-\frac{E_{\rm F} - E_{\rm V}}{kT}\right)$$

$$\therefore n p = N_{\rm V} N_{\rm C} \exp\left(-\frac{E_{\rm C} - E_{\rm V}}{kT}\right) = N_{\rm V} N_{\rm C} \exp\left(-\frac{E_{\rm g}}{kT}\right)$$

$$E_{\rm V}$$



Intrinsic (i.e., undoped) semiconductors



17, jswu

 $p = N_v e^{-(E_F - E_V)/kT}$

 $\mathbf{n} \cdot \mathbf{p} = \mathbf{N}_{C} \mathbf{N}_{V} \mathbf{e}^{-\mathbf{E}g/kT}$

 $= n_{i}^{2}$

Forbidden band

OOOOOOOOOO Filled valence band

◆ $n = p \equiv n_i$, where n_i is called *intrinsic carrier concentration* (本質載子濃度). $\Rightarrow n \cdot p = n_i \cdot n_i = n_i^2$

But, an intrinsic semiconductor is nondegenerate. (See its E_E to be calculated below.) \Rightarrow n·p = N_CN_V e^{-E_g/kT} $n = N_C e^{(E_F - E_C)/kT}$

- \therefore The above two equations give $n_i = (N_C N_V)^{1/2} e^{-E_g/2kT}$.
- $n = p \implies N_C e^{(E_F E_C)/kT} = N_V e^{-(E_F E_V)/kT}$
 - :. Intrinsic Fermi level (本質費米準位) is given by $E_{\rm F} (\equiv E_{\rm Fi} \equiv E_{\rm i}) = \frac{1}{2} \cdot (E_{\rm C} + E_{\rm V}) + \frac{1}{2} \cdot kT \cdot \ln(N_{\rm V}/N_{\rm C})$ $\approx \frac{1}{2} \cdot (E_C + E_V)$





Extrinsic (i.e., doped) semiconductors



 $n = N_C e^{(E_F - E_C)/kT}$

 $p = N_{\rm V} \; e^{-(E_F - E_V)/kT}$

 $\mathbf{n} \cdot \mathbf{p} = \mathbf{N}_{C} \mathbf{N}_{V} \mathbf{e}^{-E_{g}/kT}$

 $= n_{i}^{2}$

(N_D: donor concentration (施體濃度) & N_A: acceptor concentration (受體濃度)) If $N_{\rm D}$ or $N_{\rm A}$ is not very high, giving the semiconductor nondegenerate, then we have:

- ◆ Complete ionization (完全離子化): as shown in the figure below.
- If doped with N_{D} (>> n_{i}) \Rightarrow n-type Majority carrier (多數載子) $\mathbf{n} \approx \mathbf{N}_{\mathbf{p}} \Rightarrow \mathbf{E}_{\mathbf{F}} - \mathbf{E}_{\mathbf{C}} \approx \mathbf{kT} \cdot \mathbf{ln}(\mathbf{N}_{\mathbf{p}}/\mathbf{N}_{\mathbf{C}})$ Minority carrier (少數載子) $p = N_v e^{-(E_F - E_V)/kT}$ or $p = n_i^2/n$
- If doped with $N_A (>> n_i) \Rightarrow p$ -type Majority carrier (多數載子) $\mathbf{p} \approx \mathbf{N}_{\mathbf{A}} \Rightarrow \mathbf{E}_{\mathbf{F}} - \mathbf{E}_{\mathbf{V}} \approx \mathbf{kT} \cdot \mathbf{ln}(\mathbf{N}_{\mathbf{V}}/\mathbf{N}_{\mathbf{A}})$ Minority carrier (少數載子) $n = N_C e^{(E_F - E_C)/kT}$ or $n = n_i^2/p$



Summary of formulas (for both intrinsic & extrinsic cases)



• For a doped semiconductor: If n type $\Rightarrow n \approx N_D \& p = n_i^2/n$ If p type $\Rightarrow p \approx N_A \& n = n_i^2/p$



Symbols for undoping & doping concentration



Examples: n^+ -Si, p^{++} -Si, n-GaAs, p^- -GaAs, i-GaAs

- N_D: Donor concentration (施體濃度)
- NA: Acceptor concentration (受體濃度)

N _D or N _A	Degenerate ?	N-type	P-type
$> 10^{19} \text{ cm}^{-3}$	Degenerate	<i>n</i> ⁺⁺	$p^{\scriptscriptstyle ++}$
$10^{18} \sim 10^{19} \text{ cm}^{-3}$	~ Degenerate	n ⁺	p^+
$10^{17} \sim 10^{18} \text{ cm}^{\text{-}3}$	Nondegenerate	п	р
$10^{15} \sim 10^{17}$	Nondegenerate	n ⁻	<i>p</i> ⁻
0 (Intrinsic)	Nondegenerate	i	i

Drift (漂移)

- ♦ When *E* = 0, carriers undergo <u>random thermal motion</u> only
 ⇒ Zero net displacement over a sufficiently long period of time.
- ◆ Average thermal velocity (平均熱速度), v_{th} v_{th}~10⁷ cm/s @ 300 K (More precisely: ½m*v_{th}² = 3.½kT)
- Mean free path (平均自由路徑), l
 l = v_{th} · τ_c (~1000 Å @ 300 K)
- Mean free time (平均自由時間), or *Collision time* (碰撞時間), or *Relaxation time* (鬆弛時間) τ_c = l /v_{th} (~10⁻¹² s = 1 ps @ 300 K)
- When *C*≠0, carriers have combined motion due to random thermal motion and an applied electric field.
 ⇒ Non-zero net displacement, so-called *drift*.





22, jswu

21, jswu

Drift Velocity (漂移速度) and Mobility (遷移率)

- ◆ Drift velocity (漂移速度) $\mathbf{v} = \mathbf{\mu} \cdot \mathscr{C}$, where \mathscr{C} is the electric field. Mobility (遷移率) $\mathbf{\mu} \equiv (\mathbf{q} \cdot \boldsymbol{\tau}_{c})/\mathbf{m}^{*}$, where \mathbf{m}^{*} is the effective mass of the carriers. (f·t = m·v ⇒ $-q\mathscr{C} \cdot \boldsymbol{\tau}_{c} = \mathbf{m} \cdot \mathbf{v} \Rightarrow \mathbf{v} = -(\mathbf{q}\boldsymbol{\tau}_{c}/\mathbf{m})\mathscr{C}$)
- In most of the semiconductors, electrons have higher mobility than holes (i.e., $\mu_n > \mu_p$) due to their smaller effective mass.
- Under low fields, drift velocity is much smaller than thermal speed (i.e., $v_d \ll v_{th}$).
- A trajectory such as this would take place only under very high fields.



Drift current (漂移電流)

- *Drift current density* (漂移電流密度): J = J_n + J_p
 J = J_n + J_p = nqv_n + pqv_p = q(n·µ_n+p·µ_p) *8* ≡ σ· 8
- ◆ *Conductivity* (導電係數 or 導電率): $\sigma = q(n \cdot \mu_n + p \cdot \mu_p)$ (in Ω⁻¹·cm⁻¹) - for n-type (n >> p) ⇒ $\sigma \approx q \cdot n \cdot \mu_n$
 - for p-type (p >> n) $\Rightarrow \sigma \approx q \cdot p \cdot \mu_p$
- *Resistivity* (電阻係數 or 電阻率): ρ = 1/σ (in state)
 - $(in \Omega \cdot cm)$





28, jswu

 $q = 1.6 \times 10^{-19} C$

- Electron flow

Current flow

- Hole flow

Current flow

26, jswu



◆ Real drift velocity is represented by an <u>empirical expression</u> (經驗式)

 $\mathbf{v} = \boldsymbol{\mu}_0 \mathscr{C} / \left[1 + (\boldsymbol{\mu}_0 \mathscr{C} / \mathbf{v}_s)^{\gamma} \right]^{1/\gamma} \quad \leftarrow \text{ for Si \& Ge}$

- where μ_0 is the *low-field mobility* (低電場遷移率),
 - vs is the saturation velocity (飽和速度), and
 - γ is a factor (2 for electrons; 1 for holes).



Thermionic Emission (熱離發射)







29, jswu

Tunneling (穿隧)

◆ In quantum mechanics: *Transmission coefficient* (傳輸係數) $T = \begin{bmatrix} 1 & (qV_0 \sinh(\beta d))^2 \end{bmatrix}^{-1}$

$$T = \left[1 + \frac{(qV_0 \operatorname{sim}(pa))}{4E(qV_0 - E)}\right]$$

where $\beta = \sqrt{2m_n(qV_0 - E)}/\hbar$

• If $\beta \cdot d \gg 1$ (i.e., $V_0 \uparrow$, or $d \uparrow$, or $E \downarrow$) $\Rightarrow T \approx e^{-2\beta d} = e^{-2d\sqrt{2m_n(qV_0-E)}/\hbar}$

 \Rightarrow T is small

♦ If we want T ↑, we need d ↓, $qV_0 \downarrow$, E ↑, $m^* \downarrow$



(b)

(a) Band diagram of a semiconductor potential barrier with a distance *d*.(b) Schematic representation of the wave function across the potential barrier.

30, jswu