2.86 (a) $P[A \cup B] = P[A] + P[B] - P[A \cap B] = p_A + p_B - p_A p_B$

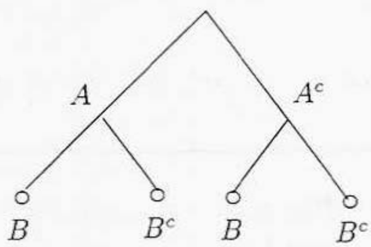
(b) $P[A \cup B] = P[A] + P[B] = p_A + p_B$

(c) $P[A \cup B \cup C] = P[A] + P[B] + P[C] - P[A \cap B] - P[A \cap C] - P[B \cap C] + P[A \cap B \cap C]$

$= p_A + p_B + p_C - p_A p_B - p_A p_C - p_B p_C + p_A p_B p_C$

(d) $P[A \cup B \cup C] = p_A + p_B + p_C$

2.87 We use a tree diagram to show the sequence of events. First we choose an urn, so A or A^c occurs. We then select a ball, so B or B^c occurs:



Now A and B are independent events if

$$P[B|A] = P[B]$$

But

$$P[B|A] = P[B] = P[B|A]P[A] + P[B|A^c]P[A^c]$$

$$\begin{aligned} \Longrightarrow P[B|A](1 - P[A]) &= P[B|A^c]P[A^c] \\ \Longrightarrow P[B|A] = P[B|A^c] & \quad \text{prob. of } B \text{ is the same given } A \text{ or } A^c\text{, that is,} \\ & \quad \text{the probability of } B \text{ is the same for both urns.} \end{aligned}$$

2.6 Sequential Experiments

2.94

(a) $P[0 \text{ or } 1 \text{ error}] = (1-p)^{100} + 100(1-p)^{99}p \qquad p = 10^{-2}$

$= 0.3660 + .3697$

$= 0.7357$

(b) $p_R = P[\text{Retransmission Required}] = 1 - P[0 \text{ or } 1 \text{ error}] = 0.2642$

$P[M \text{ transmissions in total}] = (1-p_R)\,p_R^{M} \qquad M = 1, 2, \ldots$

$P[M \text{ or more transmissions required}] = \sum_{j=M}^{\infty}(1-p)\,p_R^{j} = p_R^{M}\sum_{j=0}^{\infty}(1-p)\,p_R^{j}$

$= p_R^{M}$

2.95 Let k be the number of defective items in a batch of n tested items, then k has binomial probabilities with parameters n and $p = \frac{1}{10}$. Using Corollary 1, we have:

$$
\begin{aligned}
P[k > 1] &= 1 - P[k \le 1] = 1 - P[k=0] - P[k=1] \\
&= 1 - (1-p)^n - n(1-p)^{n-1}p \qquad p = \frac{1}{10} \\
&= 1 - \left(\frac{9}{10}\right)^n - n\left(\frac{9}{10}\right)^{n-1}\left(\frac{1}{10}\right)
\end{aligned}
$$

You should compare this problem to problem 2.76. How do the problems differ in the assumption they make?

2.96 $p = \text{prob. of success} = \frac{95}{100} = \frac{19}{20}$.

Pick n so that $P[k \geq 10] \geq 0.9$

$$P[k \geq 10] = \sum_{k=10}^{n} \binom{n}{k} p^k (1-p)^{n-k}$$

for

$$n = 11 \quad P[k \geq 10] = 0.89811$$
$$n = 12 \quad P[k \geq 10] = 0.98093 \quad \Rightarrow \text{pick } n = 12$$

2.97

(a) $P[\text{1 of } n \text{ terminals transmits}] = n(1-p)^{n-1}p$

(b) Take derivative with respect to p:

$$0 = -n(n-1)(1-p)^{n-2}p + n(1-p)^{n-1}$$

$$\Rightarrow (n-1)p = (1-p) \Rightarrow np = 1-p+p \Rightarrow p = \frac{1}{n}$$

(c) $P_{success} = n\left(1-\frac{1}{n}\right)^{n-1}\frac{1}{n} = \left(1-\frac{1}{n}\right)^{n-1} \to e^{-1} = \frac{1}{e}$ as $n \to \infty$.

$= 0.3678$

2.98

a) $P[k \text{ errors}] = \binom{n}{k} p^k(1-p)^{n-k}$

b) Type 1 errors occur with problem $p\alpha$ and do not occur with problem $1-p\alpha$

$$P[k_1 \text{ type 1 errors}] = \binom{n}{k_1}(p\alpha)^{k_1}(1-p\alpha)^{n-k_1}$$

c) $P[k_2 \text{ type 2 errors}] = \binom{n}{k_2}(p(1-\alpha))^{k_2}(1-p(1-\alpha))^{n-k_2}$

d) Three outcomes: type 1 error, type 2 error, no error

$$P[k_1, k_2, n-k_1-k_2] = \frac{n!}{k_1!k_2!(n-k_1-k_2)!}(p\alpha)^{k_1}(p(1-\alpha))^{k_2}(1-p)^{n-k_1-k_2}$$

2.99

~~2.78~~ a)

$$\begin{aligned} P[k=0] &= p \\ P[k=1] &= (1-p)p \\ P[k=2] &= (1-p)^2 p \\ P[k=3] &= 1 - P[k=0] - P[k=1] - P[k=2] = (1-p)^3 \end{aligned}$$

b)

$$\begin{aligned} P[k] &= (1-p)^k p \quad 0 \le k < m \\ P[m] &= 1 - \sum_{k=0}^{m-1} P[k] \\ &= 1 - \sum_{k=0}^{m-1}(1-p)^k p \\ &= 1 - p\frac{1-(1-p)^m}{1-(1-p)} = (1-p)^m \end{aligned}$$